Acta Oeconomica 73 (2023) 2, 275–284
DOI: 10.1556/032.2023.00016

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A simple estimation of the longevity gap and redistribution in the pension system

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Received: November 25, 2021 • Revised manuscript received: March 12, 2022 • Accepted: March 22, 2022

ABSTRACT

It has been known for decades that in a given year and in a given country, with the rise in lifetime income, life expectancy also rises. The difference between the richest and the poorest stratas' life expectancies is called *the longevity gap*. Recently, as the gap has generally been growing, it has received more and more attention. The issue is important in itself, but it has also an obvious impact on redistribution in the pension system: the greater the longevity gap, the greater is the redistribution from the low benefit pensioners to the high benefit ones in a given pension system. Econometrically estimating the life expectancy-income function may help the analysis. In our short study, first we give a simple estimation, and then we show the influence of the estimate on the redistribution.

KEYWORDS

life expectancy, lifetime income, longevity gap, redistribution in the pension system

JEL CLASSIFICATION INDICES

H55, I14

1. INTRODUCTION

While the secular rise of average life expectancy has received much attention, its heterogeneity with respect to lifetime-income has only been discovered lately. The latter phenomenon has

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been referred to as *longevity gap*. This can be quantified as the difference between the life expectancy (LE) of the richest and the poorest parts of the population (*longevity gap*).

For example, using the data of the Social Security Administration of the USA, Chetty et al. (2016) estimated this relation for the total USA population between 2001 and 2014.[1] One of their most important observations (see their Figure 2) is that among those males who survived 65, the richest 1% lives 15 years longer than the poorest percentile; for females, this gap is equal to 10 years. In that paper a table supplements their Figure, providing the relevant numbers for percentiles 20, 40, 60 and 80% without presenting the average number of longevity years for the whole population. Our related calculations witnessing this heterogeneity are displayed in Table 1, Figures 1 and 2 of the present paper. The digital version of the cited paper contains the full data set.[2] The definition and the calculation of the life expectancy at a given age are quite challenging issues but we skip them here.

The classical approach has neglected the existence of the gap and confined its attention to the issue on the dependence of the benefit-earning-ratio on the household earning: *a priori* progressivity. For example, in the US Social Security, the normal monthly pension benefit of a person is an increasing concave piecewise linear function of his/her primary monthly income.[3] Recently the impact of longevity gap on *a posteriori* redistribution in the pension system has also received increasing attention – the gap decreases the progressivity discussed in the classical approach. The greater the gap, the greater is the redistribution from the low-paid to the high-paid. In fact, the so-called proportional benefits are not proportional at all (Liebmann 2002; Whitehouse – Zaidi 2008; Holzmann et al. 2020.)

The redistribution in the pension system can be measured by the relative standard deviation of the lifetime net contribution balances across the population or aggregates of population. (Note that the standard deviation does not distinguish between the positive and negative signs

Table 1. The statistical indicators of the data set 2001–2014, US

Variables	Minimum	Average	Maximum	Standard deviation
Female LE65 (yr)	78.53	84.93	88.91	2.17
Female relative income	0.0044	0.97	20.29	2.03
Male LE65 (yr)	72.37	81.17	87.19	3.32
Male relative income	0.0036	1.03	19.68	2.10

Source: Chetty et al. (2016).

[1]https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4866586/. Haan et al. (2020) observed similar phenomena on the German cohorts.

[2]Bíró et al. (2021) documented the same gap on Hungarian data.

[3]For a related concept of progressivity index of pensions, see OECD 2011 (136–137). https://www.oecd-ilibrary.org/docserver/pension_glance-2011-24-en.pdf?expires=1637505284&id=id&accname=guest&checksum=A4E0A84468675C29607DEF5254C2586B.

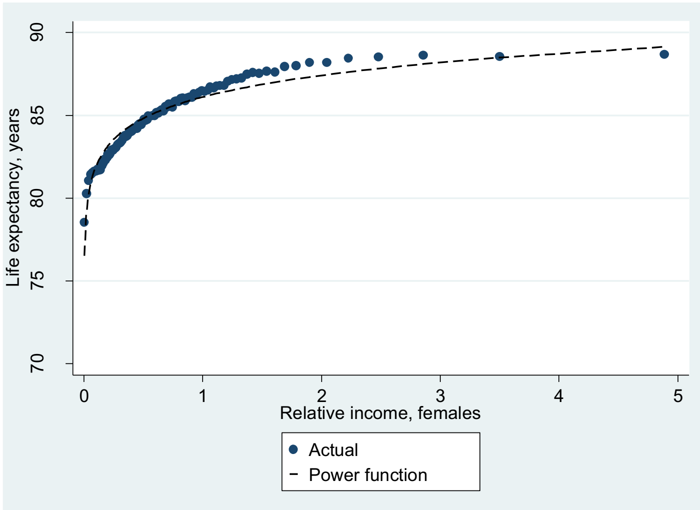

Fig. 1. Life expectancy – relative income, females

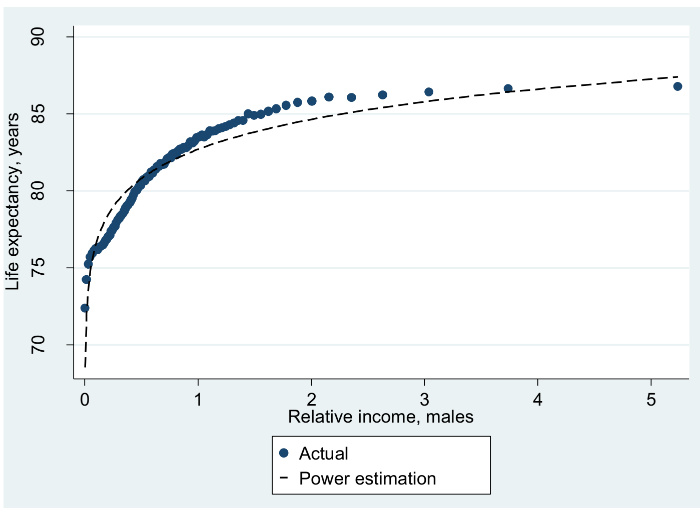

Fig. 2. Life expectancy – relative income, males

of the balance, only its absolute value matters. Therefore, the indicator shows the same way transfer from the poor to the rich and vice versa.) To measure redistribution, it is more appropriate to use absolute or relative incomes than percentile incomes that are available (Chetty et al. 2016). Holzmann et al. (2019: 323–324 and Figure 14.7) estimated the relation of life expectancy – lifetime income in quadratic and logarithmic forms. Their final conclusion:

If the estimation of redistribution is quite good for the largely heterogeneous US population, where the longevity gap is quite large and *ex ante* redistribution is quite strong, the approximation is even better in other, less heterogeneous countries, where the longevity gap is smaller and *ex ante* redistribution is weaker.[4]

In certain empirical but especially in theoretical models, it can be more suitable to use econometrically estimated functions rather than empirical data. For example, in their well-calibrated theoretical model, Sheshinski – Caliendo (2021) determined the decrease in the genuine progressivity of the US Social Security for subsequent year-groups by econometric estimations. We also proceed in this direction. Our main result is as follows: if the relation of life expectancy – lifetime income is well estimated, then the redistribution in the pension system is also correctly estimated.[5]

In our research we follow the method of Sheshinski – Caliendo (2021) (also Chapter 14 of Simonovts 2018) but use the data of Chetty et al. (2016). To eliminate two genders, we add up the male and female balances. The female – male distinction could be crucial because females live much longer, they earn much less, and they frequently enjoy widow's pensions. Its inclusion, however, would be immature at this stage.

We find that the classical assumption of constant life expectancy is unacceptable, but the power function estimate is acceptable. In addition, in another paper, Simonovits (2021) mathematically analyses the connection of the longevity gap and redistribution.[6]

Though it is a toy model, it has some policy relevance: the existence of sizable *ex post* redistribution in *a priori* neutral pension system weakens the case for nonfinancial defined contribution (NDC) public pensions (Holzmann et al. 2019).

The structure of the remainder of the paper is as follows. Section 2 presents the data in Chetty et al. (2016) and estimates the power-function form of the life expectancy-lifetime income relation. Section 3 displays the simplest version of the impact of these estimations on the redistribution. Section 4 draws the conclusions. An Appendix contains the complementary detailed statistics of our estimations.

2. DATA AND ESTIMATIONS

To present our estimations of the dependency of life expectancy at 65 years on lifetime income, it is suitable to calculate it with *relative* lifetime incomes. Since the public pension systems are unisex, but female life expectancy is much longer than males', we shall measure these incomes with the help of a common denominator. Let Y_i be the gender i's income variable (for the population disaggregated for 100 percentiles), Y_i^* be the corresponding *average* income: i = F(emale), M(ale). We take the common unweighted average income as $Y^* = (Y_F^* + Y_M^*)/2$,

[4]Sanchez-Romero et al. (2020, 2021) analysed the feedback of pension reforms on the life expectancies in a model calibrated by Austrian data. They used the internal rate of return rather than lifetime balance to compare the treatment of various strata.

[5]To avoid any misunderstanding, we emphasise that in our paper, the econometric estimation is used only for analytical purposes.

[6]It is worth mentioning that Knell (2018) studied a related but not identical problem: the impact of increasing life expectancy on national defined contribution (NDC) pension systems.

implying the relative income variable of gender i as $y_i = Y_i/Y^*$. We need the following power-function estimate:

$$\log L_i = \log L_{0i} + a(i)\log y_i, \text{i.e.}, L_i = L_{0i}y_i^{a(i)}, \qquad i = \mathrm{F, M}; \tag{1}$$

where L_i stands for the life expectancy of gender i with relative lifetime income y_i, while $L_{0\mathrm{i}}$ and $a(i)$ denote nonnegative coefficients in (1), respectively.

The descriptive statistical indicators of the data set are given in Table 1 above. Working with households, male and female incomes hardly differ.

We have already discussed the longevity gap, spanning between male and female life-expectancies of higher- and lower-paid strata, respectively, but we have only seen the tremendous lifetime income differences. The estimated parameter values of the life expectancy-income connection is displayed in Table 2, with the power function. Because we calculate across percentiles, the number of elements in the samples is, $n = 100$. We take the average values for years 2001–2014. In the Appendix, we present the detailed results of the calculations, while here we show only the main results.

In Table 2 we find that: a) R^2 is above 0.9; b) the power coefficients are equal to 0.022 (0.034), i.e., a rise of 1% relative income would raise life expectancy by 0.022 (0.034) % for females (males).

Figures 1 and 2 show the power function estimation as well as the actual data for female and male populations, respectively.[7] How does the (full) life expectancy at 65 depend on the relative lifetime incomes? Considering that only relative incomes of percentiles 99 and 100 are higher than 4, the estimations are satisfactory.

3. REDISTRIBUTION IN THE PENSION SYSTEM

It is an important question how the actual and the estimated longevity gaps influence the redistribution in the pension systems. We illustrate it in a simplest model. We assume away some important dimensions of the US and other public pension systems.

Typically pensions depend on wages, in formula $b(w_i)$ be the annual public pension benefit, approximated as a convex linear combination of proportional and flat benefits:

Table 2. Power-function estimates of the lifetime – relative income connection, 2001–2014

Coefficients	Females			Males		
	Estimates	SD	R^2	Estimates	SD	R^2
			0.92			0.90
$\log L_{0i}$	4.445	0.001		4.415	0.0017	
a_i	0.022	0.0015		0.034	0.0027	

[7]Data for percentile 100 are not indicated in the figures.

$$b(w_i) = \beta[\alpha\ w_i + (1-\alpha)], \text{where } \alpha \text{ and } \beta \text{ are real numbers between 0 and 1; } i = \mathrm{F}, \mathrm{M}, \quad (2)$$

where α showing the earnings-proportional pension share, its numerical value can be 0.5; β stands for the average replacement ratio, e.g., 0.4.[8]

We assume that everybody starts working at age $Q = 25$ and after working $S = 40$ years, she/he retires at age $R = 65$. A worker of gender i with lifetime income (normalized to annual one) y_i dies at age $L_i\ (y_i)$, i.e., spends $T_i\ (y_i) = L_i\ (y_i)–R$ years in the pension system, called duration, and i = F, M. Introducing the contribution rate τ and the duration-to-contribution length-ratio $u_i\ (y_i) = T_i\ (y_i)/S$, called relative duration, the corresponding annual balance is given by:

$$z_i(w_i, y_i) = \tau\ w_i - b(w_i)u_i(y_i), i = \mathrm{F}, \mathrm{M}. \quad (3)$$

Life expectancy depends on incomes enjoyed both before and after retirement, but the length of the latter is proportional to the expected time spent in retirement. For expositional simplicity, we normalize lifetime earnings to the annual ones. Before proceeding, we have to derive the gender-specific relation between lifetime income (y_i) and pre-retirement income (w_i), where i = F, M. Assuming that the impact of private savings (in both mandatory and voluntary pillars) on lifetime is neutral, lifetime income is the weighted average of preretirement income and public pensions, where the weight of the latter is the relative duration:[9]

$$y_i = w_i + u_i(y_i)b_i. \quad (4)$$

Depending on how $u_i(y_i)$ is estimated, it has three versions: (i) constant duration denoted by u_i(C), (ii) power function approximation, denoted by u_i(P), and (iii) actual duration, denoted by (A).

$$u_i(j) = (L_i(j) - R)/S, i = \mathrm{F}, \mathrm{M}, \text{and } j = \mathrm{C}, \mathrm{P} \text{ and } \mathrm{A}. \quad (5)$$

Substituting Eqs (2) and (5) into the implicit Eq. (4) yields:

$$y_i = w_i + u_i\ (j)\beta[\alpha w_i + 1-\alpha]. \quad (6)$$

Solving for w_i yields the wage-income-relation:

$$w_i(y_i, j) = \{y_i - u_i(j)\beta(1-\alpha)\}/\{1 + u_i\ (j)\beta\alpha\}. \quad (7)$$

Substituting (7) into (3) implies:

$$z_i[y_i] = \tau\ w_i\ (y_i) - b(w_i\ (y_i))u_i(y_i, j), i = \mathrm{F}, \mathrm{M}. \quad (8)$$

In our simplified model, we assume that the population share of both genders are equal to 1/2–1/2. We also assume that the unified two subsystems are balanced, i.e., denoting by **E** the operator of expectation over the total population in a given period, $0.5\mathbf{E}\ (z_\mathrm{F} + z_\mathrm{M}) = 0$, hence (8) implies the *balanced* contribution rate (at the given parameters):

$$\tau^*(j) = 0.5\ \mathbf{E}\ [b(w_\mathrm{F}(y_i))u_\mathrm{F}(y_\mathrm{F}, j)] + 0.5\ \mathbf{E}\ [b(w_\mathrm{M}(y_i))u_\mathrm{M}(y_\mathrm{M}, j)]. \quad (9)$$

The degree of redistribution of the pension system can be measured by the relative standard deviation (or its square, the variance) of the balances with a balanced contribution rate:

[8]The impact of the first parameter was studied in Simonovits (2021) in detail.

[9]This approach was implicitly suggested by one of the referees.

$$\sigma_{z*}^2\ (j) = \mathbf{E}\, {z^*}^2(j) \Big/ \left[\mathbf{E}\ (0.5 w_{\mathrm{F}}(y_{\mathrm{F}}, j) + 0.5 w_{\mathrm{M}}(y_{\mathrm{M}}, j)\right]^2, \text{where } z_i^*[y_i, j] \\ = \tau^* w_i(y_i, j) - b(w_i(y_i, j))u_i(y_i, j), \\ i = \mathrm{F}, \mathrm{M} \text{ and } j = \mathrm{C}, \mathrm{P}, \mathrm{A}. \tag{10}$$

We calculate the degree of redistribution for three specifications for the single period of 2001–2014: (i) classical specification: $L(y) = \mathbf{E}L$ (i.e., constant life expectancy); (ii) power function specification (1) or equivalently (5) and (iii) the actual data.

One can calculate the characteristics of the classic case analytically (on paper by pencil):

$$T = (T_{\mathrm{F}} + T_{\mathrm{M}})\Big/2, u_{\mathrm{F}} = T_{\mathrm{F}}\Big/S, u_{\mathrm{M}} = T_{\mathrm{M}}\Big/S;\ u = (u_{\mathrm{F}} + u_{\mathrm{M}})\Big/2; \tau^* = \beta u \text{ and } z_i^*(w_i) \\ = (1- \alpha)\beta\ [w_i - 1]u_i.$$

Here $\tau^* = \beta u$ is the reformulation of the following well-known identity: the balanced contribution rate is equal to the product of the replacement rate and the old-age dependency ratio. Hence,

$$\sigma_{z*}^2\ (\mathrm{C}) = (1- \alpha)\beta\ \left[u_{\mathrm{F}}^2\sigma_{w\mathrm{F}}^2(\mathrm{C}) + u_{\mathrm{M}}^2\sigma_{w\mathrm{M}}^2(\mathrm{C})\right]\Big/2,$$

where $\sigma_{w\mathrm{F}}(\mathrm{C})$ and $\sigma_{w\mathrm{M}}(\mathrm{C})$ stand for the female and male relative earnings' standard deviations, respectively. In words: the variance of the pension balances is equal to share $1- \alpha$ of the basic pensions multiplied by the average replacement ratio multiplied by the average weighted variance of earnings variances. Calculating with $L = 83$ years, the duration is equal to $T = 83 - 65 = 18$ years, the relative duration is given by $T/S = 0.45$; implying the balanced contribution rate $\tau^* = 0.4 \times 0.45 = 0.18$. (The current US contribution rate is much lower, 0.126 – but that is unsustainable.)

For $\alpha = 1$, ${\sigma_{z^*}}^2 = 0$; for $\alpha = 1/2$, the variance of the balances:

$${\sigma_{z^*}}^2 \Big/ (\mathbf{E}w)^2 = 0.2\mathrm{x}\left[0.5^2 \mathrm{\ x\ } 0.92^2 + 0.4^2 \mathrm{\ x\ } 0.98^2\right] = 0.078.$$

Table 3 shows the results of the three specifications for $\alpha = 1/2$, including the classic case above, the two other specifications need computer, the results are displayed in rows 2 and 3.

Table 3 reveals that the balanced contribution rate moderately rises with the perfection of the estimation: from 0.18 to 0.178 and 0.176. The relative standard deviation of the estimated and of the actual balances jumps from 0.078 to 0.096, and 0.130. The underlying cause of the latter is that if the life expectancies were independent of the incomes, then a 50–50% mix of proportional

Table 3. Life expectancy relative earnings specifications, contribution rates and standard deviation of the balances

Life expectancy Relative earnings $L_{\mathrm{F}}(y_{\mathrm{F}})$, $L_{\mathrm{M}}(y_{\mathrm{M}})$	Balanced contribution rate τ^*	Standard deviation of balances $\sigma_{z^*}/\mathbf{E}w$
Constant (classic)	0.180	0.078
Power function	0.178	0.096
Actual	0.176	0.130

and basic pensions would achieve a significant redistribution, the relative standard deviation of the balances would be equal to 0.078. With the existence of the gap, however, this redistribution further increases: the finer the estimation of $L_i(y_i)$ and consequently $u_i(y_i)$, the more it increases.

A fuller model would not stop here. There we should take into account the impact of redistribution on the workers' labour supply. In addition, we should consider the *welfare effects* of the pension system, i.e., how the private savings modify the impact of the contribution and the benefits on young- and old-age consumption and the impact of the redistribution on life expectancy (Miglino et al. 2023). This is beyond the scope of the present paper.

4. CONCLUSIONS

Using the US data set of Chetty et al. (2016), our short paper econometrically estimated the dependence of male and female life expectancy on relative lifetime income for the percentiles of the population. Replacing the classical approach of constant life expectancy, we have also used the power function estimation, and this way we have obtained a satisfactory estimation. Using a unisex data set and a simple pension model, we compared the balanced contribution rates and the standard deviations of the lifetime balances for three approaches. We have found that the classical approach is unacceptable, while the power-function approach is acceptable. The existence of the longevity gap increases *a posteriori* redistribution.

During the calculations, we neglected a number of complications: we skipped over the differences between lifetime earnings and total lifetime incomes; we replaced 15 cohorts by a unified group; we replaced heterogeneous contribution lengths and retirement ages by single values. We streamlined the US benefit rules (neglecting the maximization of the number of assessed years of contribution, the piecewise linearity of the benefit rule and the actuarial reduction/increase due to early/late retirement). In any further work, these simplifications must be eliminated.

ACKNOWLEDGEMENTS

This research was supported by NKF 129078. We express our debt to Anikó Bíró, Zsombor Cseres-Gergely, Gábor Kőrösi, Gábor Tusnády and anonymous referees of earlier versions.

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Appendix

Statistical additions

Dependent variable: ln(LE65)	**Females Estimated parameters**	**Males Estimated parameters**
$\ln(y/y^*)$	0.022^{***}	0.034^{***}
st.error	0.0015	0.0027
conf. interv.	0.019–0.025	0.028–0.039
constant	4.455^{***}	4.415^{***}
st.error	0.0010	0.0017
conf. interv.	4.453–4.458	4.417–4.418
aR^2	0.925	0.90
RMSE	0.0071	0.013
F	$F(1, 98) = 210.8$	$F(1, 98) = 152.9$
Number of observations	100	100

Note: RMSE: Root Mean Square Error.
Significance level: *** <1, ** <5, and *< 10%.