AKADÉMIAI KIADÓ

UNIVERSITY of DEBRECEN

International Review of Applied Sciences and Engineering

13 (2022) 3, 272–277

DOI: 10.1556/1848.2022.00368

ORIGINAL RESEARCH PAPER

Temperature evolution in heated premises and external boundary wall with dynamic control of heating systems

László Garbai and Gergely Pacza*

Department of Building Services and Process Engineering, Budapest University of Technology and Economics, Budapest, Hungary

Received: August 31, 2021 • Accepted: January 9, 2022
Published online: April 4, 2022

ABSTRACT

The study presents a mathematical model for building heating control. The buildings are connected to district heating or to central heating. The task of the heating control is to maintain a preset constant indoor air temperature. Control disturbance is caused by external meteorological conditions, firstly by outdoor air temperature. The control action can be the change in heat transfer capacity of the radiators, whereby the indoor air temperature can be commanded back to the present value to offset the effect of the disturbance. Just the control can be a follower or of predictive type. The expected indoor air temperature can be calculated from the energy balances. These are composed of differential equations, describing dynamic equilibrium of heat transfer through the external walls, as well as heat storage in the walls and indoor air. Type of differential equations is linear, inhomogeneous, of first or second order.

Solution of the differential equation results in describing the change in indoor air temperature in time as a function of outdoor air temperature. Further on, the equation determines the function of the necessary heating capacity to keep the indoor air temperature constant. By the model several heating programs can be evaluated. The physical model is shown in Fig. 1. Intermediate variable is the average temperature of the external walls. Heat conduction and convection through the walls is calculated by the difference of the indoor air temperature and of the average wall temperature by using relevant R_1 and R_2 heat resistance factors. The model is adequate when the heat transport within the building sections is neglectable, and the thermodynamic and heat transfer characteristics of the walls are identical. Thereby the model describes the heat balance of representative premises, but the results can be transferred to similar other premises, too.

KEYWORDS

sistrict heating system, predictive control, unsteady, Fourier-equation

1. INTRODUCTION

The control of heating systems is still static these days. It is based on the calculation of so-called steady operating points assigned to outdoor air temperatures. The relationship between interventions and outputs is described with an empirical function, i.e., what input characteristics are necessary for the required indoor air temperature in a steady state at a given outdoor air temperature. Previous studies have also delved into the question of what indoor air temperature is formed if there is a change or disturbance in the inputs compared to the theoretically required values and what correction of the inputs is necessary to restore the commanded indoor air temperature. In this paper, we describe a transient description method that can be applied to model the effect of changes in outdoor air temperatures over time, the thermal storage capacity of walls, and the damping effect. Thereby, we can determine the time course of the intervention characteristics as precisely as possible by which the commanded value of the indoor air temperature can be kept with the smallest deviation. The logic of the control is as follows: as a result of the change in outdoor air temperature, the change in heat loss occurs only delayed in the air of the dwelling, intervention from the

*Corresponding author.
E-mail: pacza@epget.bme.hu

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primary side, increase or decrease in heat input has to reach the air of the dwelling by this time. This mode of control obviously and demonstrably results in energy savings. Due to this dynamic control we could minimize the temperature fluctuation in the wall structure. We can reduce the thermal dilatation which can occur in structural stresses [1–4].

2. ENERGY BALANCES OF THE HEATED PREMISES. ONE AND TWO STORAGE MODELLING OF THERMAL INERTIA OF AN EXTERNAL WALL STRUCTURE

The thermal inertia of the external wall structure, the description of the unsteady heat conduction in the wall can be obtained most accurately by the Fourier equation for the multilayer wall and its numerical solution. One of the graphical methods is called Schmidt editing. The temperature distribution in the wall is theoretically continuous, we disregard to define it. Instead, we apply concentrated parameter description. The parameters under study include indoor air temperature and the average external wall temperature. If we disregard the description of the heat stored in the indoor air and study only the change in heat stored in the external boundary wall, it is called the single-storage model, otherwise a two-storage model. In our study, we write the differential equations of both single and two-storage modelling [5–7].

2.1. One storage model

We set up a model for the evolution of the average temperature of the external boundary wall and for the calculation of the heat flux through the external boundary wall during the cooling of the bounded space (heated premises) as a function of time. We defined two tasks:

- Mark the average wall temperature as tm. Our aim is to determine how the heat flux through the external boundary wall changes in the event of a sudden change in the outdoor air temperature, from t_{a1} to t_{a2}, i.e., to what extent we need to increase the input heat flux if we intend to keep the indoor air temperature at constant value. [8, 9, 11]
- The outdoor temperature changes according to an arbitrary time function, namely decreases in the model. Consequently, the average temperature of the external boundary wall decreases as well, assuming the consistency of the indoor air temperature. This decrease in the average temperature of the external boundary wall is described by Equation (1), which results in the increase of the heat flux through the wall compared to the stationary initial state (Equation 2). As a result of the increased heat flux, the indoor air temperature must necessarily decrease, for the calculation of which another differential equation is set and solved at the end of Chapter 2.1.2 [10, 12, 13]

2.1.1. Indoor air temperature change according to jump function of outdoor air temperature. A further advantage of the control is that we can minimize the temperature fluctuations in the boundary structures (external walls) and therefore the degree of thermal dilatation and stresses. We can reduce the risk of damage to the building structure and increase the lifetime of the building [11, 12, 14].

During the research, two tasks were defined as follows:

- Average wall temperature test, and
- Outdoor air temperature test

The geometric model of control is of paramount importance. The heated rooms in the building require constant indoor air temperature. In our model, we test how heat flux through the boundary walls changes with outdoor air temperature and how much heating power is needed to keep the indoor air temperature constant. The geometric model is shown in Fig. 1 [15, 16].

The following equations can be applied for the heat flux leaving the wall:

$$\dot{q}_1 = \frac{t_i - t_m}{R_1} \tag{1}$$

$$\dot{q}_2 = \frac{t_m - t_a}{R_2} \tag{2}$$

Afterwards, we write the change in the amount of heat in the wall per unit of time, which will be equal to the difference in the increased thermal flux due to the decrease in the initial and outdoor air temperatures, so

$$c_w M \frac{dt_m}{d\tau} = \dot{q}_1 - \dot{q}_2 = \frac{t_i - t_m}{R_1} - \frac{t_m - t_a}{R_2} \tag{3}$$

in an ordered way

$$c_w M \frac{dt_m}{d\tau} = \dot{q}_1 - \dot{q}_2 = \frac{t_i}{R_1} - \frac{t_m}{R_1} - \frac{t_m}{R_2} + \frac{t_a}{R_2} \tag{4}$$

$$c_w M \frac{dt_m}{d\tau} = \left(\frac{t_i}{R_1} + \frac{t_a}{R_2}\right) - t_m\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{5}$$

In equation (5), t_i is constant.

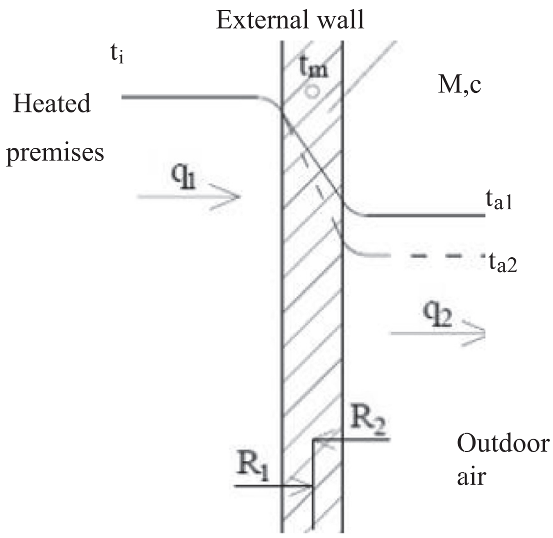

Fig. 1. Heat transfer model for heated premises

Constants A and B are introduced.

$$c_w M \frac{dt_m}{d\tau} = A - t_m B \tag{6}$$

where,

$$A = \left(\frac{t_i}{R_1} + \frac{t_a}{R_2}\right) \text{és}\, B = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Divided by the thermal capacity of the wall and introducing constants A * and B *

$$\frac{dt_m}{d\tau} = \frac{A}{c_w M} - t_m \frac{B}{c_w M} \tag{7}$$

The obtained differential equation is a constant coefficient, linear, first order, homogeneous, which is a separable variable. After separating the variables, the differential equation can be solved by direct integration.

The ordered form of the differential equation by introducing new constants

$$\frac{dt_m}{d\tau} = A^* - t_m B^* \tag{8}$$

where,

$$A^* = \frac{A}{c_w M} \text{és}\, B^* = \frac{B}{c_w M}$$

In an ordered way,

$$\frac{dt_m}{A^* - t_m B^*} = d\tau, \tag{9}$$

Marking integration,

$$\int \frac{1}{A^* - t_m B^*} dt_m = \int 1 d\tau\,. \tag{10}$$

Performing integration,

$$-\frac{1}{B^*} ln\left(A^* - t_m B^*\right) = \tau + C, \tag{11}$$

With further order,

$$\left(A^* - t_m B^*\right) = Ce^{-B^* \tau} \tag{12}$$

Finally,

$$t_m = \frac{A^*}{B^*} - \frac{C}{B^*} e^{-B^* \tau} \tag{13}$$

The value of the integration constant C can be determined from the initial condition, according to which the average temperature of the wall at the initial time is a given initial value, i.e., if

$$\tau = 0; \quad t_m = t_{m,0}, \tag{14}$$

$$t_{m,0} = \frac{A^*}{B^*} - \frac{C}{B^*}\,, \tag{15}$$

Replacing it into Equation (13), we get a formula describing the change in the mean wall temperature as a function of time:

$$t_m = \frac{A^*}{B^*} - \frac{\left(A^* - t_{m,0} B^*\right)}{B^*} e^{-B^* \tau}, \tag{16}$$

heat flux from heated premises

$$\dot{q}_1(\tau) = \frac{t_i - t_m}{R_1} = \frac{t_i}{R_1} - \frac{1}{R_1}\left[\frac{A^*}{B^*} - \frac{\left(A^* - t_{m,0} B^*\right)}{B^*} e^{-B^* \tau}\right] \tag{17}$$

If t_i is constant as commanded value, than radiator power must be equal $\dot{q}_1(\tau)$.

2.1.2. Indoor air temperature change in the case of change of outdoor air temperature according to a continuous time function. The test model corresponds to paragraph 2 and Fig. 1, however, as a fine-tuning of the model, the outdoor air temperature t_a now does not change sharply, but changes according to some time function $t_a\,(\tau)$. Discussing the tested phenomenon in this way is closer to reality and gives more accurate results.

As above, thermal flow heat flux can be written.

$$\dot{q}_1 = \frac{t_i - t_m}{R_1} \tag{18}$$

$$\dot{q}_2 = \frac{t_m - t_a}{R_2} \tag{19}$$

$$t_a = t_a(\tau), \tag{20}$$

$$c_w M \frac{dt_m}{d\tau} = \dot{q}_1 - \dot{q}_2 = \frac{t_i - t_m}{R_1} - \frac{t_m - t_a(\tau)}{R_2} = \frac{t_i}{R_1} - \frac{t_m}{R_1} - \frac{t_m}{R_2} + \frac{t_a(\tau)}{R_2} \tag{21}$$

After ordering

$$c_w M \frac{dt_m}{d\tau} = \left(\frac{t_i}{R_1} + \frac{t_a(\tau)}{R_2}\right) - t_m\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{22}$$

Introducing notations A, B, and D, it can be written that

$$c_w M \frac{dt_m}{d\tau} = A - t_m B + D t_a(\tau), \tag{23}$$

where

$$A = \left(\frac{t_i}{R_1}\right),\ B = \left(\frac{1}{R_1} + \frac{1}{R_2}\right),\ D = \left(\frac{1}{R_2}\right).$$

Divided by the thermal capacity of the wall,

$$\frac{dt_m}{d\tau} = \frac{A}{c_w M} - t_m \frac{B}{c_w M} + \frac{D}{c_w M} t_a(\tau) \tag{24}$$

Introducing the modified notations A *, B * and D *, we obtain a linear, first-order, inhomogeneous differential equation with a constant coefficient.

$$\frac{dt_m}{d\tau} = A^* - t_m B^* + D^* t_a(\tau) \tag{25}$$

where,

$$A^* = \frac{A}{c_w M},\ B^* = \frac{B}{c_w M},\ D^* = \frac{D}{c_w M}.$$

The canonical form of the equation,

$$y' + X(x)y = -X_1(x), \tag{26}$$

where,

$$X(x) = B^*, \tag{27}$$

$$X_1(x) = A^* + D^* t_a(\tau)$$

The solution is as follows,

$$y = e^{u(x)}\left[-\int X_1(x)e^{-u(x)}dx + C_1\right] \tag{28}$$

where,

$$u(x) = -\int X(x)dx = -\int B^* dx = -B^* x\,. \tag{29}$$

Therefore,

$$y = t_m(\tau) = e^{-B^* x}\left[-\int \left(A^* + D^* t_a(\tau)\right)e^{B^* x}d\tau + C_1\right] \tag{30}$$

After ordering, we get the differential equation describing the change of the average temperature of the wall, in the case of the change of the outdoor temperature according to the time function

$$t_m(\tau) = e^{-B^* x}\left[-\int A^* e^{B^* x}d\tau - D\int t_a(\tau)e^{B^* x}d\tau + C_1\right] \tag{31}$$

Further simplified,

$$t_m(\tau) = e^{-B^* x}\left[-\frac{A^*}{B^*}e^{B^* x} - D\int t_a(\tau)e^{B^* x}d\tau + C_1\right] \tag{32}$$

Of which, the change indoor air temperature can be calculated with equation (18).

2.2. Two heat storages model for calculation of indoor air temperature

In this subsection, we test transient processes in a system composed of indoor air and external wall structure. Our goal is to write the differential equation describing the transient, i.e., to determine what happens between two equilibriums.

The following energy balance can be written for indoor air:

$$cm\frac{dt_i}{d\tau} = \dot{q}_{rad} - \frac{t_i - t_m}{R_1} \tag{33}$$

and the energy balances of the wall are:

$$c_w M\frac{dt_m}{d\tau} = \frac{t_i - t_m}{R_1} - \frac{t_m - t_a}{R_2} \tag{34}$$

$$c_w M\frac{dt_m}{d\tau} = \frac{t_i}{R_1} - t_m\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{t_a}{R_2} \tag{35}$$

From Equation (33):

$$t_m(\tau) = R_1 cm\frac{dt_i}{d\tau} + t_i(\tau) - \dot{q}_{rad}R_1 \tag{36}$$

Replace Equation (36) into Equation (35) to obtain the differential equation describing the change in $t_i\ (\tau)$.

$$c_w M\frac{d}{d\tau}\left[R_1 cm\frac{dt_i}{d\tau} + t_i(\tau) - \dot{q}_{rad}R_1\right] = \frac{t_i(\tau)}{R_1} - t_m(\tau)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{t_a}{R_2} \tag{37}$$

$$c_w M R_1 cm\frac{d^2 t_i}{d\tau^2} + c_w M\frac{d}{d\tau}t_i - c_w M\frac{d}{d\tau}\dot{q}_{rad}R_1 = \frac{t_i(\tau)}{R_1} - t_m(\tau)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{t_a}{R_2} \tag{38}$$

After ordering, we get equation (39),

$$c_w M R_1 cm\frac{d^2 t_i}{d\tau^2} + c_w M\frac{d}{d\tau}t_i - c_w M R_1\frac{d}{d\tau}\dot{q}_{rad} - \frac{t_i(\tau)}{R_1} = -\left[R_1 cm\frac{dt_i}{d\tau} + t_i(\tau) - \dot{q}_{rad}R_1\right]\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{t_a}{R_2} \tag{39}$$

Cases tested:

- q_{rad} = const.
- q_{rad} is a function of τ, therefore qrad = qrad (τ),

If $\dot{q}_{rad} = constant$, for simpler use, we get equation (40) by introducing notations a_0, a_1, a_2, a_3 and a_4:

$$a_0\frac{d^2 t_i}{d\tau^2} + a_1\frac{dt_i}{d\tau} + a_2 t_i = a_3 t_a(\tau) + a_4 \tag{40}$$

in which constants a_0, a_1, a_2, a_3, a_4 are as follows:

$$a_0 = c_w M R_1 cm \tag{41}$$

$$a_1 = c_w M + R_1 cm\left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$

$$a_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{1}{R_1} = \frac{1}{R_2},$$

$$a_3 = \frac{1}{R_2},$$

$$a_4 = -\dot{q}_{rad}R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

With the solution of differential equation (40) the change of indoor air temperature is obtained.

The obtained differential equation is of constant coefficient, linear, second-order, inhomogeneous, which can be solved by the method of varying the constants or by the method of probe functions.

The solution of Equation (40) by the method of varying the constants in matrix form is as follows:

We first solve the homogeneous part of the equation:

$$a_0\frac{d^2 t_i}{d\tau^2} + a_1\frac{dt_i}{d\tau} + a_2 t_i = 0. \tag{42}$$

Characteristic equation for the homogeneous part:

$$a_0\lambda^2 + a_1\lambda + a_2 = 0. \tag{43}$$

The characteristic equation can be solved by the general solution formula of the quadratic equation.

$$\lambda_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_0a_2}}{2a_0} \quad (44)$$

Solutions to Equation (43):

$$\lambda_1 = \frac{-a_1}{2a_0} + \sqrt{a_1^2 - 4a_0a_2} \quad (45)$$

$$\lambda_2 = \frac{-a_1}{2a_0} - \sqrt{a_1^2 - 4a_0a_2} \quad (46)$$

The homogeneous solution

$$y_h = c_1 e^{\lambda_1 \tau} + c_2 e^{\lambda_2 \tau} \quad (47)$$

The constants c_1 and c_2 can be determined by the initial conditions.

Solving the particular part of the equation then, assuming that constants c_1 and c_2 are also functions of time

$$y_p = c_1(\tau) e^{\lambda_1 \tau} + c_2(\tau) e^{\lambda_2 \tau} \quad (48)$$

Using the method of varying the constants, an equation of the particular part can be created in matrix form.

$$y_p = \sum_{i=1}^{n} c_i(\tau) y_i(\tau) \quad (49)$$

$$\begin{bmatrix} c_1' \\ c_2' \\ \vdots \end{bmatrix} = \begin{bmatrix} y_1 & y_2 & \cdots \\ y_1' & y_2' & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ f(\tau) \\ \vdots \end{bmatrix} \quad (50)$$

Writing Equation (47) in a matrix form:

$$\begin{bmatrix} c_1' \\ c_2' \end{bmatrix} = \begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ f(\tau) \end{bmatrix} \quad (51)$$

Replaced

$$\begin{bmatrix} c_1' \\ c_2' \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 \tau} & e^{\lambda_2 \tau} \\ \lambda_1 e^{\lambda_1 \tau} & \lambda_2 e^{\lambda_2 \tau} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ a_3 t_a(\tau) + a_4 \end{bmatrix} \quad (52)$$

When performing the matrix inversion:

$$\begin{bmatrix} c_1' \\ c_2' \end{bmatrix} = \frac{1}{e^{(\lambda_1+\lambda_2)\tau} + (\lambda_2 - \lambda_1)} \begin{bmatrix} \lambda_2 e^{\lambda_2 \tau} & -e^{\lambda_2 \tau} \\ -\lambda_1 e^{\lambda_1 \tau} & e^{\lambda_1 \tau} \end{bmatrix} \begin{bmatrix} 0 \\ a_3 t_a(\tau) + a_4 \end{bmatrix} \quad (53)$$

By performing the matrix multiplication and expressing the values of c_1 'and c_2', the constants c_1 and c_2 can be determined by integration.

$$c_1' = \frac{-e^{\lambda_2 \tau}}{e^{(\lambda_1+\lambda_2)\tau} + (\lambda_2 - \lambda_1)} (a_3 t_a(\tau) + a_4) \quad (54)$$

$$c_2' = \frac{e^{\lambda_1 \tau}}{e^{(\lambda_1+\lambda_2)\tau} + (\lambda_2 - \lambda_1)} (a_3 t_a(\tau) + a_4) \quad (55)$$

From Equation (54):

$$c_1(\tau) = \int \frac{-e^{\lambda_2 \tau}}{e^{(\lambda_1+\lambda_2)\tau} + (\lambda_2 - \lambda_1)} (a_3 t_a(\tau) + a_4) d\tau \quad (56)$$

From Equation (54):

$$c_2(\tau) = \int \frac{-e^{\lambda_2 \tau}}{e^{(\lambda_1+\lambda_2)\tau} + (\lambda_2 - \lambda_1)} (a_3 t_a(\tau) + a_4) d\tau \quad (57)$$

The solution of the initial equation (39) is the sum of the homogeneous and particular partial solutions, therefore:

$$y = y_h + y_p = c_1 e^{\lambda_1 \tau} + c_2 e^{\lambda_2 \tau} + c_1(\tau) e^{\lambda_1 \tau} + c_2(\tau) e^{\lambda_2 \tau} \quad (58)$$

Therefore the evolution of indoor air temperature

$$t_i(\tau,\ t_a) = y \quad (59)$$

This theory was successfully applied for evaluation of thermodynamic characteristics in Hungarian panel buildings. The results will be presented in a forthcoming article.

3. SUMMARY, CONCLUSIONS

The description of unsteady heat conduction in a wall can be obtained most precisely by numerical solution of system of equations consisting of Fourier equations written for a multilayer wall structure. With the models in this paper, an approximation with different accuracy can be performed. In regular building engineering practice, the calculations described here have not been used yet, but they may open the door to a new, predictive control of district heating systems. The mechanism of action of disturbance, which means a change in the outside temperature, is slow. The change of the thermal loss occurs only with a delay in the tested space, the intervention from the primary side, the increase or decrease of the heat supply or circulation only must arrive in the air of the dwelling by this time.

The formulas described in Section 2 can be used to calculate the time during which, in the event of a sudden change in the outside temperature, the change in air temperature takes place in addition to the constant heat input, which does not have a negative effect on our sense of comfort. The equation to be solved is linear, with constant coefficient, first order and homogeneous (Equation 8). The problem can be further detailed by calculating the amount of excess heat that needs to be introduced so that the required internal temperature does not decrease with the change of the outdoor temperature. Alternatively, the decrease in internal temperature over a given period can be calculated (Equation 38).

Section 3 discusses the thermal behaviour of the wall structure in the event of a change in the outside temperature as a function of time. Due to the change as a function of time, the equation thus written is already inhomogeneous (Equation 46). The equation (53) describing the change in the average wall temperature is thus much more complicated than that described in Section 2.

The combined behaviour of the system formed by the air and the wall structure was tested in Section 2. We have written a system of equations that is constant coefficient, linear, second order, and inhomogeneous. The system of equations can be reduced to a single differential equation by transformations.

We have described the theoretical course of the solution, which consists of a general solution of the homogeneous part and a particular solution of the inhomogeneous equation.

Whether higher accuracy and solution of this model undoubtedly justifies its application can be decided by further studies. Of course, the same applies to the application of the Fourier-equation, which gives the most accurate description of the transient thermal conduction processes and the definition of the temperature field in complex wall structures. Regarding the application of the Fourier-equation, it is worth mentioning that due to the complicated initial and boundary conditions, there is only a numerical solution to the problem. In addition, Fourier-equation with different characteristics must be applied to each wall layer, and a quadratic (contact) boundary condition to be written for the contact of the layers appears, according to which the temperatures at the points of contact are the same.

LIST OF NOTATION

R_1 R_2	heat transfer resistance m^2 W^{-1} K^{-1}
t_m	average temperature of external boundary wall °C
t_a	outdoor air (ambient) temperature
t_i	indoor air temperature
q_1, q_2	heat flux w/m^2
M	specific mass of external boundary wall kg m^{-2}
m	specific mass of indoor air kg m^{-2}
τ	time s
A	external boundary wall surface m^2
c_w	specific heat capacity of wall
c	specific heat capacity of air
A, B, C	constant
a_o, a_1,a_2,a_3	constant
ln	natural logarithm

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