AKADÉMIAI KIADÓ

UNIVERSITY of DEBRECEN

International Review of Applied Sciences and Engineering

15 (2024) 3, 397–412

DOI:
10.1556/1848.2024.00803

ORIGINAL RESEARCH PAPER

A complex methodology for the development of mathematical modeling skills in engineering education

Dóra Sipos and Imre Kocsis*

Department of Basic Technical Studies, Faculty of Engineering, University of Debrecen, Ótemető u. 2-4, 4028 Debrecen, Hungary

Received: January 9, 2024 • Accepted: March 19, 2024
Published online: April 16, 2024

ABSTRACT

Today, the role of humans is changing rapidly in both industrial production activities and services. Mediocre, easy-to-learn activities can be performed more efficiently by machines; mediocre knowledge is being devalued while the importance of high-level skills is increasing. As a result, in all sectors of the economy, and especially in engineering, new approaches to expert training are needed; people must learn to hand over certain decision-making roles and to control the processes supported by AI rather than compete with it. STEM education has a responsibility to achieve these goals and must develop appropriate tools for engineering education. This paper presents a complex didactic methodology for competency-based education in engineering bachelor programs. An important element is the mathematical competency map, which shows the importance and place of mathematical and algorithmic (coding) knowledge in engineering topics. Another element is the systematic testing of mathematical knowledge in non-mathematical contexts in engineering courses. We provide an overview of our achievements in applying the developed toolset and improving the efficiency of mathematics teaching in engineering bachelor programs.

KEYWORDS

engineering education, application of mathematics, mathematics in professional context, competency mapping, engineering competencies, modeling skills

1. INTRODUCTION

The competition between humans and machines has reached a decisive level with the widespread use of AI-based solutions in more and more areas of our lives.

The industrial revolutions up to Industry 4.0 were about the rise of machine solutions as opposed to the use of human resources (use of human power, repetitive movements, algorithmic processes, etc.). Although the concept of Industry 5.0 is about machine-human collaboration rather than competition, today the role of humans is changing even faster due to the rise of AI, affecting sensitive areas such as thinking, creative work, and decision-making. A typical area of rapid change is code generation. In the last decade, one of the most important and well-paid skills was general programming; in the future, the majority of routine coding tasks will be solved by software, and high-level integration will require human experts.

This means that mediocre, easy-to-learn activities can be performed more efficiently by machines, that mediocre knowledge is devalued, and that high-level fluency becomes more important. As a result, the training of experts in all sectors of the economy will require new approaches, and people will have to learn how to hand over certain decision-making roles and control AI-assisted processes. Training that focuses on easily acquired knowledge will not produce competitive professionals.

In the areas of smart building services, smart factories, social robots, and autonomous vehicles, the majority of future engineers will need to be involved in research and

*Corresponding author.
E-mail: kocsisi@eng.unideb.hu

AKJournals

development rather than simply using a technology. This means that they have to learn and use a large number of development tools at different levels of application.

Consequently, in the era of smart engineering and intelligent systems, the required engineering competencies are shifting to algorithmic thinking, control and coding skills, understanding of machine-to-machine and machine-to-human communication and collaboration, as well as some soft competencies such as communication and teamwork skills.

All problems in the world of robotics and AI ultimately boil down to understanding and manipulating mathematical models: elements, connections, mappings, classification, regression, optimization. An interesting experience of the authors in postgraduate engineering courses is that after understanding the "simple" mathematics behind machine learning, participants' thinking about intelligent machines and AI changes radically.

When machines can take over even some high-level activities from humans, the perfect determination of hard and soft engineering skills must be carefully identified and effectively transferred in engineering education. In preparation for competency-based education, competency maps are being developed that show the importance and place of mathematical and algorithmic (coding) knowledge in professional topics in mechanical engineering, mechatronics engineering, and automotive engineering bachelor programs. A demonstration of the math-competency map can be found in Appendix A.

The competency maps show that certain mathematical topics (algorithms, numerical methods, simulation) play a crucial role, while others are necessary to build a consistent mathematical mindset. At the undergraduate level, it is necessary to learn the basic concepts and applications of differential calculus, integral calculus, linear algebra, ordinary differential equations, Fourier theory, multivariable and vector functions, and statistics. For real engineering applications, even in the first semester, symbolic computations must be combined with simple numerical methods that can be used to solve simple (e.g., discrete-time) models.

In the new situation created by the need for well-educated engineers on the one hand, and the unlimited availability of information and generative AI on the other, the traditional organization of learning material in higher education is not effective enough. To optimize knowledge transfer in engineering education, a new well-organized learning process is required.

In the engineering bachelor programs of the Faculty of Engineering of the University of Debrecen a complex educational methodology has been applied in the previous years, the details of which are summarized in Chapter 2.2. A novel element of our didactic methodology presented in this paper is the mathematical topic mapping and the systematic testing of mathematical knowledge in non-mathematical contexts within the framework of engineering courses in collaboration with the lecturers of the technical subjects. On the basis of the math-competency map, the mathematical concepts and methods used in the different subjects are identified and the current knowledge of the students is tested. The results serve as feedback for the teachers of the engineering mathematics courses and draw the students' attention to the knowledge elements they need to repeat or relearn. Students who pass the tests receive bonus points in the engineering courses' grading system; the feedback is the goal, not the evaluation of the students.

2. MATERIALS AND METHODS

2.1. Literature review, motivation of the research

Although methodological research in STEM has long been associated with the teaching of mathematics and physics, in the last decade researchers have shown an increasing interest in studying the entire process of engineering education. On the one hand, the results of mathematical didactics serve as a foundation for the methodology of teaching technical subjects, and on the other hand, mathematics is one of the crucial elements of engineering education, so that questions of mathematics education naturally arise when studying engineering education. Today, the methodology of engineering education also has a significant literature. In the following, we will mention only a few publications whose subject is close to the world of thought of our article.

Study [1] explored how students can experience the relevance of mathematical modeling activities. Modeling activities were designed within a mathematics course for engineering students, which included a guest lecture from an employee of an engine company who used mathematical modeling in his job. It was found that students perceived the modeling activities as relevant, but that doing mathematics was also perceived as relevant only to get grades, leave school, and enter careers that may not require mathematics. Based on the study, according to a report of the Mathematics Working Group of SEFI, the authors offered recommendations for making mathematics education more relevant to more students. The report provides guidance for the design of teaching processes, teaching and learning environments and approaches, and also sets out concrete mathematics curricula for the specific engineering programs that will help students to acquire the competencies to an adequate degree. Only mathematics education integrated in engineering studies can provide the ability to use mathematics in engineering contexts.

In the study, mathematics was seen as a preparation for learning engineering subjects, as well as a subject that shapes one's perspective and increases one's professional intelligence. In terms of assessments in mathematics, the focus on learnable and algorithmic knowledge cannot show the real usefulness and the ability of students to apply the knowledge in a long-term, creative way.

The University of Pretoria study reported at [2] discusses the retention of mathematical knowledge and skills acquired in the first year of the training after two additional years of study. The paper focuses on the long-term retention of basic mathematical techniques in a first-year calculus course.

Although the research found that there is a significant overall decline in performance over a two-year period, there are areas in which students still performed reasonably well or even showed improvement after the elapsed period. The conclusions can help course designers determine where to emphasize different topics and skills. In nine introductory STEM courses, the practice of spaced recall of knowledge elements was studied in [3]. The authors examined the effect of revisiting certain topics over time with delays.

The results of Hungarian national surveys on the STEM competencies of students entering higher education in engineering or science fields show that a significant proportion of students start their studies with an unacceptable level of mathematical knowledge [4]. The lack of basic knowledge in mathematical thinking and computational skills is an increasing obstacle to understanding even the simplest mathematical modeling examples. Meanwhile, engineering practice requires more and more mathematical skills.

Only a new teaching approach can reflect to the recent changes in the circumstances of the engineering higher education. The difficulty of fundamental courses (referred to as “barriers” to STEM degrees in [5]) must be addressed. According to [6], the low level of achievements in mathematics subjects can be partly attributed to inappropriate teaching methods. Since the difficulties observed in engineering mathematics courses indicate low efficiency in solving engineering problems, the methodological issues of mathematics education for non-mathematics students, including engineering students, need to be brought to the forefront.

Professional competencies need to be at the forefront of engineering education in order to ensure efficient job performance, therefore engineering education should be systematically aligned with professional requirements. The research on the level of professional competencies of engineering students reported in [7] concludes that engineering education must focus on teaching activities that enable students to be professionally prepared for their future work.

2.2. Levels and elements of the educational program

Our experience shows that the efficiency of the usual process of learning the theory and doing calculations in mathematics courses and using concepts and calculation methods in engineering courses is rather low in the case of most engineering students.

A complex educational methodology has been introduced and studied at the Faculty of Engineering of the University of Debrecen, and some of our observations and experimental results have been published. The main elements of our methodology at the different levels of the engineering education are as follows.

- Level of the engineering bachelor programs:
  - modeling approach throughout the education, increasingly complex models starting with discrete-time systems [8];
  - submission of cross-course multi-semester homework projects with attractive and motivating engineering problems [8, 14].
- Level of the engineering courses using mathematics intensively:
  - mapping mathematical competences in the engineering curriculum;
  - distributed knowledge transfer, preparing special mathematics notes for technical courses, problem-based learning (in cooperation between teachers of mathematics and technical subjects) [9];
  - relearning or learning mathematical topics in engineering courses, mathematics in a technical context.
- Level of the engineering mathematics courses:
  - simultaneous discussion of analytical and numerical methods;
  - coding numerical calculation algorithms;
  - immediate feedback and continuous assessment;
  - focus on mathematical models rather than computational techniques;
  - use of a database of engineering motivated multi-level mathematical problems in mathematics classes [10];
  - use of mathematical software (preferably Matlab and Simulink) to avoid the unnecessary simplifications and to show the use of mathematical calculations in engineering problem solving;
  - completing projects (larger tasks for individuals or teams), presentations.

2.2.1. Concept image, the role of the cyclic recall of the key knowledge elements. Engineering R&D activities require a perfect understanding of certain key concepts and the ability to abstract. Our experience in entrance tests [11] shows that most first-year students have problems even with the adequate use of basic mathematical concepts. Consequently, even at the university level lecturers need to deal with the concept image which is defined in [12] and consists of all the cognitive structure in the individual’s mind that is associated with a given concept.

In traditional higher education, it was quite natural to simply present definitions, theorems, and methods and leave the students to deal with the curriculum. Nowadays, this approach can lead to serious problems regarding the outcome of the teaching process. Teachers need to take into account that students’ concept image may be quite different from the formal concept definition [12].

In the entrance examination and during the first semester of engineering mathematics, it is not only computational skills that need to be carefully assessed, but also the level of understanding of concepts. Many mathematical concepts are not formally defined in elementary and secondary schools; students learn to recognize them through experience and use in appropriate contexts [12]. Later, in higher education, the concepts are redefined and given a symbol and a name that allows them to be communicated and helps in their mental manipulation.

According to [12] the term concept image is used to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. The concept image changes when it is recalled. Appendix B shows an example how a concept is recalled during the learning process in our methodology.

For each individual, a concept definition generates its own concept image, called the "concept definition image" in [12]. Repeated recall of concepts can reveal the misconception that can cause difficulties in understanding the engineering theory. Ultimately, we need to ensure that students can fully understand the definition of the key concepts.

2.2.2. Special purpose mathematical notes, distributed knowledge transfer, problem-based learning. In competency-centred education, some mathematical topics need to be linked to applications rather than to dedicated mathematics courses, while maintaining the integrity of mathematics education.

Concentrated discussion of all mathematical topics from the simplest to the most difficult makes mathematics education stressful and unmotivating for students, while a didactically designed distribution of knowledge transfer can make mathematics interesting through thorough problem solving in engineering courses.

In our experience, simply referring to large mathematical textbooks when studying engineering topics is almost useless. Short, specialized mathematical notes must be prepared where the discussion is application-oriented.

Problem-based learning of engineering mathematics can serve the study of advanced engineering topics that require high-level mathematical knowledge and its creative application.

A case study of problem-based learning in the context of the Technical Diagnostics course in the Mechanical Engineering program was presented and a special purpose note was prepared in the context of "ThinkBS - Basic Sciences in Engineering Education, Erasmus Plus Project" [13].

Technical Diagnostics is one of the most important professional courses in the Maintenance Engineering specialization of the Mechanical Engineering bachelor program.

Since the main topic of the course is condition monitoring based on vibration measurement and vibration signal analysis, Fourier theory is an important element of the theoretical background. Although professional software used in industry provides "ready-to-use" applications, correct planning of measurements and evaluation of results is impossible without a clear understanding of the integral transforms used in vibration analysis. To deal with this situation, the parallel discussion of professional tasks and the mathematical background is necessary, and the application of a project-based approach is inevitable.

The field of technical diagnostics can be considered as applied mathematics and applied computer science. Integral transformations, filters and other signal processing techniques are used intensively, the theory of which without "physical meaning" is too abstract and consequently difficult or impossible to understand for most mechanical engineering students. However, if students can see the role and use of, for example, the frequency spectrum, and understand the connection between signal representation in the time domain and in the frequency domain, the concept of function decomposition in terms of orthogonal systems can become clearer [9].

Some basic requirements that were taken into consideration in the project are the following: a special purpose mathematical note must

- be a specific collection of mathematical concepts and methods, not a complete discussion of some mathematical topics;
- be a problem book rather than a textbook with many worked examples and exercises;
- meet the needs of students at different levels of knowledge and abstraction, starting with the basics and ending with advanced topics that show the interdependent elements of knowledge;
- present engineering applications in detail, where all calculation steps are done by the students on paper or using software;
- follow the topics of the engineering course, and the mathematical knowledge must also be checked in tests and student projects.

The chapters of the special purpose note in Technical diagnostics are

- Trigonometric and Exponential Functions
- Statistical Analysis of Vibration Signals
- Hilbert Spaces, Orthogonality, Similarity of Functions
- Orthonormal Systems, Fourier Series, Trigonometric System
- Exponential System, Vibration Spectrum
- Continuous Fourier Transform, Discrete Fourier Transform, FFT
- Cepstrum Analysis, Envelope Analysis
- Continuous and Discrete Wavelet Transform
- MRA, Scalogram
- Wavelet Transforms in Machine Fault Diagnostics
- Digital Filters, FIR, IIR
- Digital Filter Design

2.2.3. Cross-course, multi-semester homework projects. The efficiency of engineering education can be significantly increased if the students at each stage of the educational process are clear how the parts of the curriculum are connected, to what other parts they are connected, and what tasks they will be able to solve with the complex knowledge.

The projects, which give tasks to the students throughout the training, serve to provide an overview of the entire training process and to understand the learning goals. They have to solve the partial tasks at a level corresponding to their current knowledge, but they have to get usable results from the beginning.

In the mechatronics bachelor's program, a multi-semester homework project has been prepared that includes knowledge elements from mathematics, physics, computer science, and several related technical subjects. The topic of the project can be found in part at [14], which presents the investigation of the suspension of a quarter car model from the point of view of observability and controllability. An active suspension with full state feedback control is modeled and a state observer is designed. A full description of the project is under publication.

2.2.4. Immediate feedback. Among several modes of feedback built into our methodology (see Appendix B), the "real-time" immediate feedback method has the most important role in the teaching process in engineering mathematics classes. A method of immediate feedback has been presented and the results have been discussed by the authors in [15]. The primary goal of the end-of-class online surveys at the was not to evaluate the students. We wanted to assess how well the students understood the class material and how well they could recall the topics of the previous weeks. This type of feedback provides an opportunity for quick correction and also serves to deepen and solidify the new information.

2.2.5. Multi-level engineering problems in mathematics classes. A task database has been created in which the problems are divided into three categories: purely mathematical problems motivated by technical applications; technical problems for which the model is provided and only mathematical knowledge is required for the solution; technical problems formulated in a professional context that require model building and higher-level, complex mathematical knowledge. The details and the use of the tasks in engineering mathematics education are presented by the authors in a paper under review [10].

Supporting mathematics teaching with real engineering problems is inevitable today when motivation is a crucial aspect of engineering education. A typical problem with presenting applications in mathematics classes is that mathematics teachers focus only on the calculations in the mathematical model and skip the steps leading to the model. It is clear that it is better not to talk about the physical modeling process than to present the ideas in an inappropriate way. A proper and holistic presentation of some important modeling processes requires effective collaboration between teachers of mathematics, physics and other engineering subjects.

2.3. Testing and development of modelling skills

Questionnaire on first-year students' attitudes toward mathematics and their knowledge of mathematical modeling was conducted, the survey was completed by all first-year mechanical engineering students in 2022 (123 students).

The online questionnaire with the Kahoot application consisted of 27 questions. After the question about their results in high school mathematics, we asked the students about their attitude to mathematics (questions 2–16). In the first group of questions related to modeling (questions 17–22), they had to decide which of the following categories given things belong to:

- concept,
- mathematical model,
- physical model, or
- physically existing thing.

In the second group of questions related to modeling (questions 23–27), they had to decide whether statements were true or false.

Questions and response ratios can be found in Appendix D.

2.4. Testing mathematical knowledge in engineering context

As an important part of our methodology, we prepare and apply special mathematical tests to check the mathematical knowledge in an engineering context. Appendix C gives examples of mathematical tests in engineering context in three engineering subjects, namely

- electromagnetics,
- vehicle and drive elements and
- logistics.

A similar test in statics (mechanics) called 'delayed test' was mentioned by the authors in [10] where the topics were the following

- determination of coordinates of a force in different coordinate systems (the related mathematical topic is linear transforms of the plane);
- calculation of the resultant of a distributed force system from the force density function (the related mathematical topic is calculation of definite integrals);
- calculation of shear force at a cross-section of a prismatic beam from the bending moment function (the related mathematical topic is calculation of derivative functions);
- calculation of the moment vectors of given forces and determination their mutual position in the space (the related mathematical topic is scalar and vector product of special vectors).

Some primary and secondary benefits of this way of testing are shown in Table 1. The most important results are the common thinking of mathematics and engineering teachers about the relationship between the subjects and the cooperative work on competency-based educational process in different engineering majors.

The concept and methodology of testing mathematical knowledge in an engineering context is as follows.

First, the most important mathematical skills are identified in collaboration with lecturers of engineering courses. Table 2 in Appendix E shows the mathematical knowledge needed to answer the questions in three of the courses involved in this research and details about test questions in Statics can be found at [10].

Test questions are then prepared deliberately avoiding the usual mathematical terminology. We are interested in the later "life" of concepts and methods in engineering environments rather than schematic steps or visual information that students can recall from memory. In our methodology, efficiency measures the level of ability to apply in engineering practice.

Students are not informed before or during the tests about the goal of it, for them these tests are normal tests of

Table 1. The primary and secondary benefits of mathematical knowledge tests in engineering context

Primary benefits	Secondary benefits
– repeated recall of key knowledge elements that deepens the understanding – feedback on the long-term effect of mathematics instruction – verify the quality of the concept image in an engineering context – Math Competency Map – communication and collaboration between mathematics and engineering teachers	– motivational tool for math teachers – draws attention to the role of mathematical tools and thinking in engineering problem solving – generates joint educational projects – supports students' scientific activity – improves the understanding of mathematical modelling

the course. Actually these questions could appear in normal tests but they have the peculiarity that the level of knowledge related to the given engineering course is minimized allowing students to concentrate of the mathematic needed to answer the questions.

Mathematics instructors check the solutions and use them as feedback for their courses, as shown in Appendix B.

3. RESULTS

3.1. Testing and development of modelling skills

The purpose of the questionnaire was to gather information about first-year students' attitudes toward mathematics in general and their concept of a mathematical model. The questions and the distribution of responses can be found in Appendix D.

To see the relationship between secondary school performance and attitudes and thinking about mathematical modeling, we also analyzed separately the responses of the "best students" with excellent secondary school math grades and "others".

As expected, the answers of the best students were more or less different from those of the others. For comparison, we were interested in the questions where the answers were significantly different.

The majority of students (80%) believe that they can learn the mathematics curriculum, but only the teacher's explanation is enough for 44% of them and 68% of them need additional individual help.

There is a significant difference between the best and other students in the question of whether just listening to the teacher's explanation is enough for them to understand the curriculum. For 78% of the best students it is usually enough, but for only 44% of the others it is usually enough.

100% of the best students but only 17% of the others can usually or always understand what the math teacher is talking about.

Math is difficult to understand for 41% of the general population, for 16% of the best students it is usually difficult or not difficult at all, and for 49% of the others.

Among the best students, 53% don't feel bad at all when doing math, and 47% feel bad sometimes, while 22% of the others usually or always feel bad, which is a significant difference, while, surprisingly, there is no big difference in the answers related to feeling anxious during the calculations.

69% of the best students but only 36% of the others are always or usually confident before writing tests. 69% of the best students but only 22% of the others are happy or very happy to learn a lot of math.

In general, 81% of students think that mathematics is usually not far from real things, while 23% of others think that it is. 69% of the best students can clearly see the connection between mathematics and the real world and know what mathematics is for.

It is generally accepted that practical examples help to understand mathematics.

62% of students believe that learning more mathematics is really necessary and 73% believe that they will be able to use the mathematics they learn. It is noteworthy that while all the best students relied that they would be able to use it, only 65% of the others answered it.

The rate of correct classification in questions 17–22 was generally very low (see Fig. 1). It shows that for the majority of students the difference between the categories concept, mathematical model, physical model, physically existing thing is not clear. Thus, the role of mathematical modeling must be in the focus of mathematics teaching for engineering students.

Another reason for focusing on mathematical modeling is quite different. Nowadays, all calculations that appear in engineering education can be done with mathematical software or online applications. Consequently, we need to teach computational methods to support the construction of models and the understanding of the relationships between the concepts used in the models rather than to train students to do computations "on paper".

The answers to the remaining questions show that the students could answer some questions quite well, but they

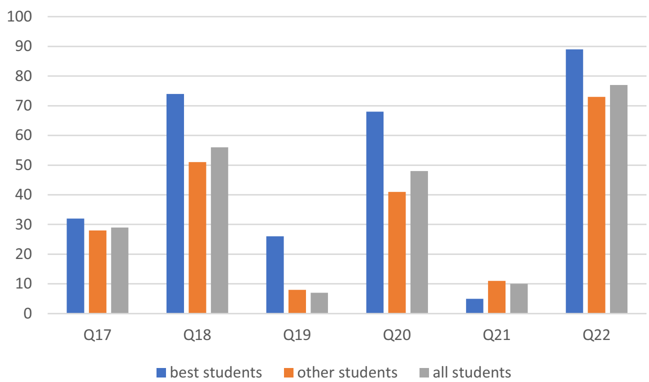


Fig. 1. Percentage of good answers to questions 17–22

are a bit confused in this area. There was no significant difference between the two groups of students (see Fig. 2).

Most students answered correctly that different mathematical models of a technical/practical problem can lead to different solutions. But they failed to understand that a mathematical model has “a life of its own”, in the course of problem solving it is an important step to check the solutions obtained in the mathematical model from engineering point of view.

Unpublished research of the Faculty of Engineering of the University of Debrecen on the modeling skills of secondary school students shows that although they generally prefer to deal with mathematical models of real-world problems rather than pure mathematical manipulations in tests, they are less successful in this type of problems due to the lack of experience and appropriate guidance.

Seeing the connection between the original problem and its physical and mathematical models is a foundation of engineering thinking. For calculations in a mathematical model, it is not necessary to understand the original technical/practical problem that led to the model. Therefore, pure mathematics can be taught in an abstract way, which is unfortunately the typical way of teaching mathematics. The low rate of good answers to question 26 shows that the mathematical model is not considered separate from the investigated problem by the students.

The solutions we can get from the mathematical model depend on the model, as mentioned above. Furthermore, mathematical models are usually derived through simplifications, so they can lead to approximate solutions of the original problem.

3.2. Testing mathematical knowledge in engineering contexts

In our practice, mathematical knowledge in engineering context has been regularly tested since 2022. This study involved 179 people (30 students in the Electromagnetics course, 45 students in the Vehicle and drive elements course, 24 students in the Logistics course, 80 students in the Statics course).

Appendix D shows the mathematical knowledge needed to answer the questions and the assessment of the level of

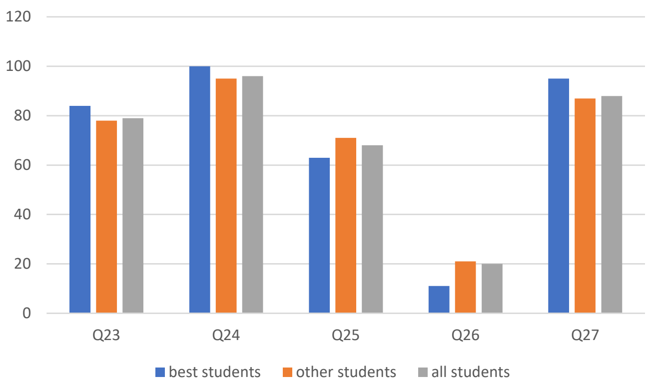


Fig. 2. Percentage of good answers to questions 23–27

knowledge on the basis of the test results. The meaning of the notations used in Table 2 is as follows

✓ = majority of students could use the related math
! = majority of students could not use the related math
o = none or almost none of the students could use the related math

If the majority of students could not apply the mathematical concept or calculation method, then the teaching of that topic needs to be revised. We need to find out whether the students could not decide which method to use, or whether they could remember the method but could not apply it. We can help students to choose the right method by presenting more types of engineering problems in mathematics class, and to perform calculations perfectly by repeating the calculation methods more times.

If none or almost none could use the mathematics involved, the students probably could not understand the question or could not see the connection between the problem posed and the mathematics they had learned. If too much time has elapsed between learning and using a mathematical tool, a special mathematical note needs to be prepared or extended with the problematic topic.

According to the results in Table 2, the students who participated in this survey had no problems with the basics. However, we can find ‘!’ or ‘o’ marks in all other columns.

Note the large number of ‘!’ or ‘o’ marks in the vector and matrix operations and differentiation columns. This may come as a surprise, since these are the easiest topics in Mathematics I, and even the weakest students can pass these questions. This phenomenon shows that application skills are not strongly correlated with the difficulty of the mathematical topic.

Students were not allowed to repeat the topics before the tests used in the research, so they could not refresh their knowledge, so most of them could not remember the details. If the routine calculations are not linked to applications, it is very difficult to keep them in mind.

An advantage of long-term, cross-curricular projects is that all the calculations are linked to the steps of a complex process and can be recalled as elements of a “story”. As in our daily lives, it is easier to remember things that appear in stories than those that appear in isolation, out of context.

Partial differentiation and integration cause many difficulties for students. The main ideas of these topics have to be relearned over and over again in subsequent courses. Our experience shows that we can do everything in mathematics courses, but we cannot ensure that students will have the knowledge and be able to use it in new contexts several semesters later. With a distributed knowledge transfer, we have a better chance of giving students usable tools.

4. DISCUSSION

Consistent with the literature in the field, our experience shows that the goals of engineering education require a well-designed teaching process with a didactic background.

Education is a complex process that requires complex methods and improvement techniques. In the traditional education only students had to adapt to the expectations in order to keep the chance to continue learning and to obtain the desired degree.

In our world, especially in modern industry, two of the key elements of success are the ability to collaborate and control, which must be supported by a cooperative approach and control in engineering education. Our methodology is based on the analysis of the educational process and the identification of the critical points where the chance of routine can yield significant improvement of the outcome.

An important thing to consider in education is that modeling skills, and especially understanding the role of mathematical models and algorithmic thinking, is a basic engineering competency in modern industry, along with some soft competencies. In this situation, certain parts of mathematics become important for all engineers involved in design and development.

Learning mathematics cannot be an isolated part of engineering education and a necessary evil. Mathematical tools must be integrated elements of the education that form the basis of effective learning. Based on the appearance of mathematical tools in different forms during the education, a synergy of knowledge transfer processes can be established. In this paper, after giving a brief overview of the achievements of the Faculty of Engineering of the University of Debrecen in improving the efficiency of mathematics teaching in engineering programs, we have introduced some new elements of our didactic methodology, first of all a way of testing mathematical knowledge in non-mathematical contexts within the framework of engineering courses. Some elements of the complex approach are Appendix B.

Our survey of freshman modeling skills shows that students have no experience in applying the cycle of 1. understanding the real problem; 2. creating a physical model of the problem; 3. creating the mathematical model of the problem; 3. finding the solutions in the mathematical model; 4. finding the solutions in the physical model; 5. finding the solution to the original problem. Consequently, the focus must be on modeling rather than on computation.

The modeling-centric approach is also supported by a large arsenal of computational tools. Due to the large gap between students' ability to abstract and the level of abstraction of engineering models, it is necessary to introduce models gradually from the beginning of the education, according to the increasing level of complexity.

The systematic use of mathematical tests in an engineering context as feedback is an efficient tool for improving the mathematics teaching process together with the use of the database of engineering problems requiring different levels of modeling in mathematics teaching. The test results clearly show the mathematical topics that cause problems for most students, and therefore need to be repeated more often in mathematics courses and mentioned in special mathematical notes prepared for engineering courses.

Collaboration between mathematics and engineering faculty based on the Math Competency Map (Appendix A) can serve as a model for reorganizing knowledge transfer in related engineering courses to make more efficient use of available time and human resources overall.

Conflict of interest: The 2nd author, Imre Kocsis is a member of the Editorial Board of the journal. Therefore, the submission was handled by a different member of the editorial team.

REFERENCES

[1] P. Hernandez-Martinez and P. Vos, ""Why do I have to learn this?" A case study on students' experiences of the relevance of mathematical modelling activities," *ZDM*, vol. 50, pp. 245–57, 2018.

[2] J. Engelbrecht, A. Harding, and J. Du Preez, "Long-term retention of basic mathematical knowledge and skills with engineering students," *Eur. J. Eng. Educ.*, vol. 32, no. 6, pp. 735–44, 2007. https://doi.org/10.1080/03043790701520792.

[3] C. R. Bego, K. B. Lyle, J. C. Immekus, and P. A. Ralston, "Introducing desirable difficulty in STEM barrier courses with spaced retrieval practice," in *2021 IEEE Frontiers in Education Conference (FIE)*, IEEE, 2021, pp. 1–6.

[4] A. Csakany, "Results of mathematics 'test zero'at Budapest university of technology and economics in 2010," *Pollack Period.*, vol. 7, nos Supplement-1, pp. 43–53, 2012. https://doi.org/10.1556/pollack.7.2012.s.4.

[5] A. Redmond-Sanogo, J. Angle, and E. Davis, "Kinks in the STEM pipeline: tracking STEM graduation rates using science and mathematics performance," *Sch. Sci. Maths.*, vol. 116, no. 7, pp. 378–88, 2016. https://doi.org/10.1111/ssm.12195.

[6] K. B. Lyle, C. R. Bego, R. F. Hopkins, J. L. Hieb, and P. A. Ralston, "How the amount and spacing of retrieval practice affect the short-and long-term retention of mathematics knowledge," *Educ. Psychol. Rev.*, vol. 32, pp. 277–95, 2020. https://doi.org/10.1007/s10648-019-09489-x.

[7] A. D. Plutenko, A. V. Leyf, A. V. Kozyr, and T. V. Haletskaya, "Specific features of vocational education and training of engineering personnel for high-tech businesses," *Eur. J. Contemp. Educ.*, vol. 7, no. 2, pp. 360–71, 2018. https://doi.org/10.13187/ejced.2018.2.360.

[8] I. Kocsis, R. Mikuska, C. Budai, K. A. Kis, and P. Korondi, "Discrete-time modeling and animation with mechatronics approach for control education," in *2023 IEEE 10th International Conference on E-Learning in Industrial Electronics (ICELIE)*, IEEE, 2023, pp. 1–6.

[9] I. Kocsis and D. Sipos, "On research and training in machinery diagnostics in engineering education," *Műszaki Tudományos Közlemények*, vol. 17, no. 1, pp. 31–6, 2022.

[10] D. Sipos and I. Kocsis, "On a method for measuring the effectiveness of mathematics teaching using delayed testing in technical contexts in engineering education," in *Annales Mathematicae et Informaticae*, accepted manuscript, 2024

[11] Gy. Szanyi and A. Varga, "Matematikai alapok a mérnökképzésben: kereslet és kínálat," in *Proceedings of the Conference on Problem-based Learning in Engineering Education*, Debrecen, University of Debrecen Faculty of Engineering, 2018, pp. 70–7.

[12] D. Tall and S. Vinner, “Concept image and concept definition in mathematics with particular reference to limits and continuity,” *Educ. Stud. Math.*, vol. 12, no. 2, pp. 151–69, 1981.

[13] I. Kocsis and D. Sipos, “Project-based learning in technical diagnostics course in a mechanical engineering bachelor programme, ThinkBS basic sciences in engineering education, Erasmus Plus project,” in *International Workshop 25–26 March*, 2021, Istanbul, Turkey, Abstract Book, 2021, p. 22. http://thinkbsworkshop.com/.

[14] K. A. Kis, Gy. Korsoveczki, K. Sarvajcz, P. Korondi, I. Kocsis, and I. Balajti, “Quarter car suspension state space model and full state feedback control for real-time processing,” in *2023 Signal Processing Symposium (SPSympo)*, IEEE, 2023, pp. 73–8. 6 p. 2023.

[15] D. Sipos and I. Kocsis, “Supporting the education of engineering mathematics using the immediate feedback method,” *Teach. Maths. Comput. Sci.*, vol. 21, no. 1, pp. 49–61, 2023.

Appendix A

	basic algebra	functions	vector operations	matrix operations	differentiation	integration	Vectorfields	Fourier theory
mechanics	✓	✓	✓	✓	✓			
electromagnetics		✓	✓		✓	✓	✓	
logistics	✓	✓						
vehicle and drive elements	✓	✓	✓					
technical diagnostics		✓			✓	✓		✓

Fig. 3. A demonstration of the Math Competency Map (a part of the Map)

Appendix B

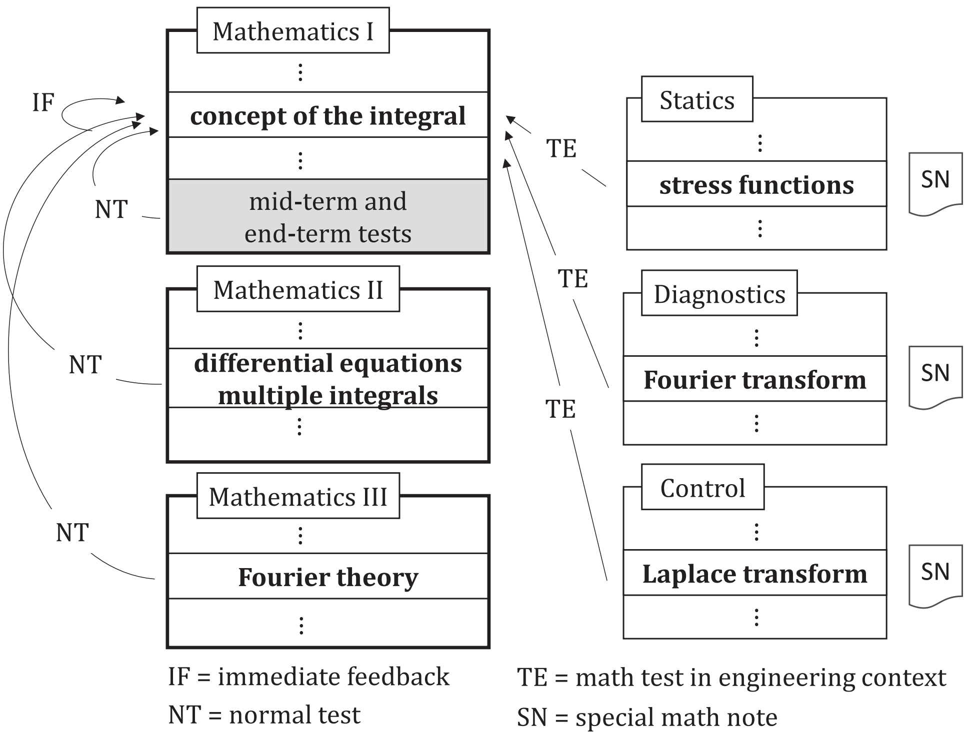


Fig. 4 Recalling and testing the key elements of mathematical knowledge.

Appendix C

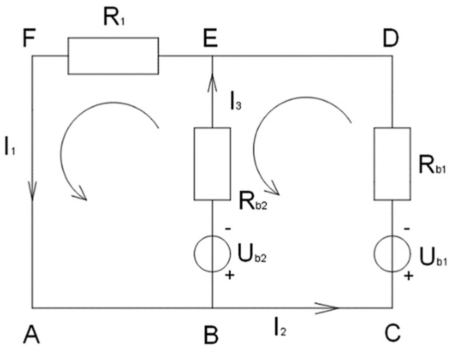
Examples of mathematical tests in engineering context

C.1 Electromagnetics

Problem EM-1

Apply Kirchhoff second law to loops $A-B-E-F-A$ and $B-C-D-E-B$ of the DC network given in the figure and apply Kirchhoff's first law to node B.

Determine the currents.



$U_{b_1} = 5$ [V]
$U_{b_2} = 20$ [V]
$R_1 = 2$ [Ω]
$R_{b_1} = 5$ [Ω]
$R_{b_2} = 2$ [Ω];

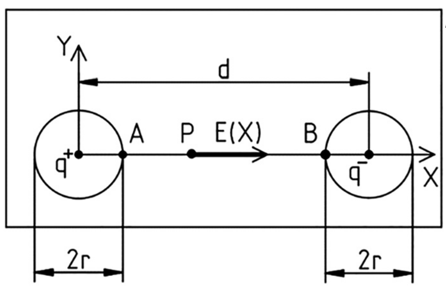
Problem EM-2

The figure shows a conventional car battery from the top.

Calculate the voltage between the two electrodes if the magnitude of the field strength between two electrodes depending on the location is given by function.

$$E(x) = k \cdot q \cdot \left(\frac{1}{x^2} + \frac{1}{(d-x)^2} \right), x \in [0, d]$$

Use formula $U_{AB} = \int_r^{d-r} E(x)dx$.

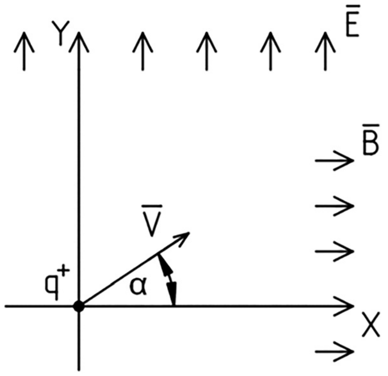


$k = 8,988 \cdot 10^9 \left[\frac{N \cdot m^2}{c^2}\right]$
$q = 7 \cdot 10^{-12} [c]$
$d = 0,3 [m]$
$r = 0,01 [m]$

Problem EM-3

The positively charged particle q moves with speed $\overline{v}$ in the $x-y$ plane, where a homogeneous electric field parallel to the x-axis and a homogeneous magnetic field parallel to the y-axis are present.

Give the force on the particle. Use formula $\overline{F} = q \cdot (\overline{E} + \overline{v} \times \overline{B})$.



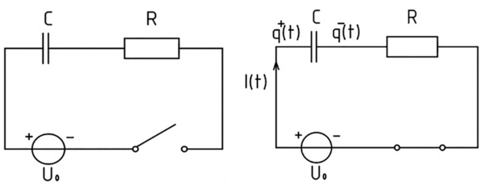
$q = 7 \cdot 10^{-6}$ [C]
$E = 4 \cdot 10^6 \left[\frac{N}{C}\right]$
$B = 2$ [T]
$v = 5 \cdot 10^6 \left[\frac{m}{s}\right]$
$\alpha = 30°$

Problem EM-4

In the circuit shown in the figure, a capacitor of capacity C is charged after the switch is closed.

Determine the strength of the charging current as a function of time, if the charge of the capacitor as a function of time is given by the formula

$$q(t) = C \cdot U_0 \cdot \left(1 - e^{-\frac{t}{R \cdot C}}\right), t > 0$$

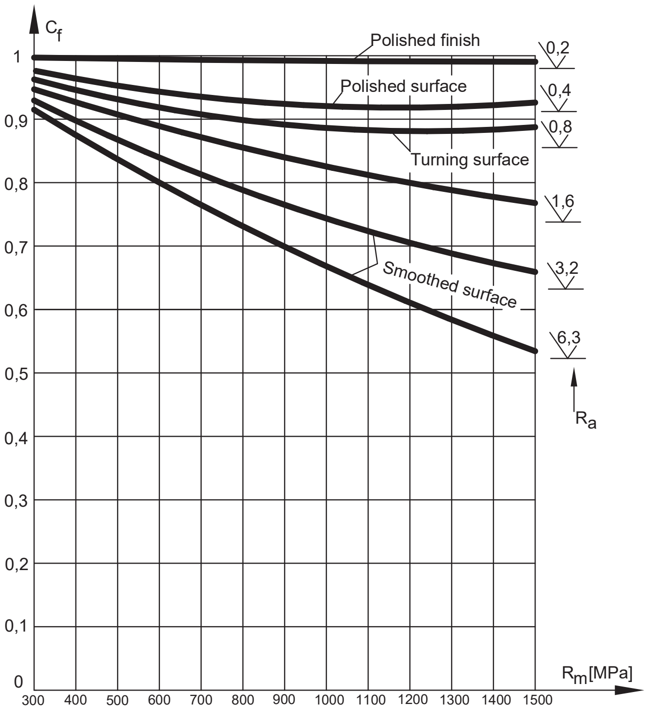


$U_0 = 10$ [V]
$C = 10^{-4}$ [F]
$R = 10^5$ [Ω]

C.2 Vehicle and drive elements

Problem VD-1

With the help of the diagram, calculate the value of the surface factor C_f, if the surface roughness $R_a = 6,3$ and the surface tension is 1840 MPa. Supposed that the surface tension over 600 MPa is linear.



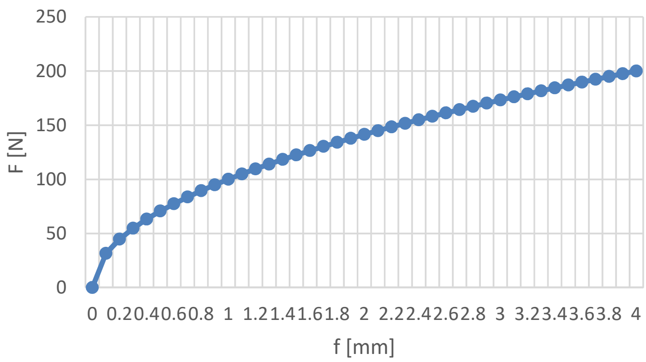
Problem VD-2

Calculate the thread height angle for a single M8 screw.

The size of the M8 screw: $P = 1.25$, $d_2 = 7.188$, $d_3 = 6.466$

Problem VD-3

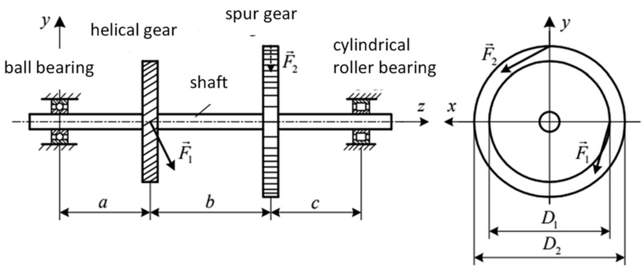
Calculate the work of the following progressive characteristic spring from $0 - 4$ mm compression.
The following points are given
0 mm – 0 N
1mm – 100N
4mm – 200N



Problem VD-4

Calculate the supporting force at points A and B.

The drive shaft loading diagram:



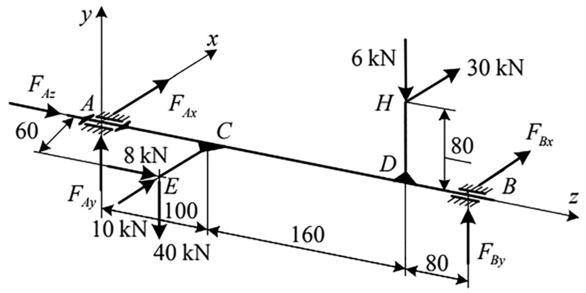
Length dimensions: $a = 100\ mm, b = 160\ mm, c = 80\ mm$

Rolling circle diameters of the gears: $D_1 = 120\ mm$, $D_2 = 160\ mm$

Forces acting on the gears: $\overrightarrow{F}_1 = (10\overrightarrow{e}_x - 40\overrightarrow{e}_y + 8\overrightarrow{e}_z)kN$, $\overrightarrow{F}_2 = (30\overrightarrow{e}_x - 6\overrightarrow{e}_y)kN$

Steps of the solution:

The mechanical model:



Give the supporting forces:

Moment equation on point A:

$$\overrightarrow{M}_A = \overrightarrow{r}_{A1}\times\overrightarrow{F}_1 + \overrightarrow{r}_{A2}\times\overrightarrow{F}_2 + \overrightarrow{r}_{AB}\times\overrightarrow{F}_B = \overrightarrow{0}\ \ /\cdot\overrightarrow{e}_x\ /\cdot\overrightarrow{e}_y$$

$$\overrightarrow{r}_{A1} = \left(-0,06\overrightarrow{e}_x + 0,1\overrightarrow{e}_z\right)m,\ \overrightarrow{r}_{A2} = \left(0,08\overrightarrow{e}_y + 0,26\overrightarrow{e}_z\right)m,\ \overrightarrow{r}_{AB} = \left(0,34\overrightarrow{e}_z\right)m$$

$$\overrightarrow{r}_{A1}\times\overrightarrow{F}_1 =$$

$$\overrightarrow{r}_{A2}\times\overrightarrow{F}_2 =$$

$$\overrightarrow{r}_{AB}\times\overrightarrow{F}_B =$$

Give the scalar equations:

Moment equation on Point B:

$$\overrightarrow{M}_B = \overrightarrow{r}_{B1}\times\overrightarrow{F}_1 + \overrightarrow{r}_{B2}\times\overrightarrow{F}_2 + \overrightarrow{r}_{BA}\times\overrightarrow{F}_A = \overrightarrow{0}\ \ /\cdot\overrightarrow{e}_x\ /\cdot\overrightarrow{e}_y$$

$$\overrightarrow{r}_{B1} = \left(-0,06\overrightarrow{e}_x + 0,24\overrightarrow{e}_z\right)m,\ \overrightarrow{r}_{B2} = \left(0,08\overrightarrow{e}_y - 0,08\overrightarrow{e}_z\right)m,\ \overrightarrow{r}_{BA} = \left(-0,34\overrightarrow{e}_z\right)m$$

$$\overrightarrow{r}_{B1}\times\overrightarrow{F}_1 =$$

$$\overrightarrow{r}_{B2}\times\overrightarrow{F}_2 =$$

$$\overrightarrow{r}_{BA}\times\overrightarrow{F}_B =$$

Give the scalar equations:

C.3 Logistics

Problem L-1

The receipt of orders has nominal distribution, the expected value is 100, the standard deviation 5. The inventory cost of one product is 1 EUR, in case of shortage 2 EUR have to be paid. Calculate the size of the economically optimal safety stock!

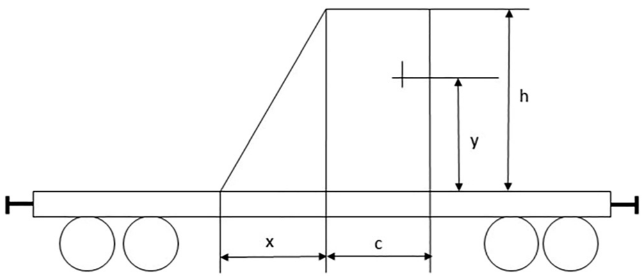
How does the result change if the expected value is only 20?

Do the calculation with shortage cost of 5 EUR as well!

Problem L-2

A railway wagon which has a weight of 11 tons which carries a machine lasts 10 tons (stored in a wooden box) which was fixed with inclined fastening. It bumps with a speed of 5 km h^{-1} into a parking 12 tons weight railway wagon, which carries a 10 tons weight machine in a skid box.

The freight and the fixing parameters of the overrun railway wagon are the following:
$c = 1,5\ m, y = 1,3\ m, h = 2,5\ m, x = 1,1\ m.$
Collision parameters are
$c_p = 0,43 \cdot 107 \frac{N}{m}, c_w = 5 \cdot 107 \frac{N}{m},$
$F_{coll.} = 320\ kN$



Problem L-3

Calculate the unit packaging costs including the logistic costs of a product which has a yearly consumption of 151, 000 pcs. A customer orders the product in average for 48 weeks. The unit package contains 6 pcs, 1 pallet contains 15 pcs of unit package. The transportation cost per unit transport takes 115 EUR, one truck can carry 52 pallets. The average utilization of the truck is 80%. The warehouse handling cost takes 0.98 EUR/unit package.

Problem L-4

The demand function of a product is $f(p) = 150 - 3p$, the offer function is $S(p) = 2p - 20$.

The unit price is in EUR, the quantity in pcs.

Draw the Marshall cross and define the equilibrium point (the equilibrium price and the equilibrium quantity).

Problem L-5

There are two types of computer working in an office: C and D. Computer C is operating daily for c hours, computer D for d hours. The daily performance is described by the function

$$f(c; d) = 18c + 20d - 2c^2 - 4d^2 - cd$$

which gives the number of the tested programs on a day.

It is also known that none of the computers are allowed to work more than 5 hours a day.

Determine how many hours should the computers operate optimally if we want to achieve the highest possible performance.

Appendix D

Questionnaire on first-year students' attitudes toward mathematics and their knowledge of mathematical modeling

Q1. What was your math grade average in high school?

Choose the answer that best describes you in the given questions.

Q2. I can learn the math curriculum

not at all/sometimes/usually yes/in all cases

Responses

	not at all	sometimes	usually yes	in all cases
best students	0%	10%	74%	16%
other students	0%	22%	73%	5%
all students	0%	20%	73%	7%

Q3. It is enough for me to listen to the teacher's explanation to understand the curriculum

not enough at all/sometimes enough/usually enough/always enough

Responses

	not enough at all	sometimes enough	usually enough	always enough
best students	0%	22%	68%	10%
other students	21%	44%	35%	0%
all students	16%	40%	43%	1%

Q4. I need to ask for individual help to understand the curriculum

I always ask for help/usually I ask for help/sometimes I ask for help/I never ask for help

Responses

	I always ask for help	usually I ask for help	sometimes I ask for help	I never ask for help
best students	0%	10%	58%	32%
other students	0%	22%	71%	7%
all students	0%	20%	68%	12%

Q5. I can learn the curriculum on my own

I can never learn it on my own/sometimes I can learn it on my own/usually I can learn it/I can always learn it on my own

Responses

	I can never learn it on my own	sometimes I can learn it on my own	usually I can learn it	I can always learn it on my own
best students	0%	10%	74%	16%
other students	3%	30%	67%	0%
all students	2%	26%	68%	4%

Q6. Math is hard to understand

not difficult at all/sometimes difficult/usually difficult/always difficult

Responses

	not difficult at all	sometimes difficult	usually difficult	always difficult
best students	10%	74%	16%	0%
other students	2%	49%	46%	3%
all students	4%	55%	39%	2%

Q7. I feel bad when I have to do math

I don't feel bad at all/sometimes I feel bad/usually I feel bad/I always feel bad

Responses

	I don't feel bad at all	sometimes I feel bad	usually I feel bad	I always feel bad
best students	53%	47%	0%	0%
other students	25%	53%	16%	6%
all students	32%	51%	12%	5%

Q8. I feel fear during the reckoning

I always feel fear/sometimes I feel fear/I usually feel fear/I never feel fear

Responses

	I always feel fear	sometimes I feel fear	I usually feel fear	I never feel fear
best students	6%	74%	10%	10%
other students	19%	50%	14%	17%
all students	16%	55%	13%	16%

Q9. I am usually confident before writing papers

I'm always confident/I'm usually confident/sometimes I'm confident/I'm never confident

Responses

	I'm always confident	I'm usually confident	sometimes I'm confident	I'm never confident
best students	6%	63%	25%	6%
other students	8%	28%	36%	28%
all students	7%	37%	34%	22%

Q10. I usually don't understand what the math teacher is talking about

I always understand/I usually understand/I rarely understand/I never understand

Responses

	I always understand	I usually understand	I rarely understand	I never understand
best students	47%	53%	0%	0%
other students	4%	79%	16%	1%
all students	14%	73%	12%	1%

Q11. Mathematics is far from the real things

not far away/usually not far away/far away/very far away

Responses

	not far away	usually not far away	far away	very far away
best students	69%	25%	6%	0%
other students	37%	40%	20%	3%
all students	44%	37%	17%	2%

Q12. I don't know what math is for

I don't know at all/I don't care/I usually know/I know

Responses

	I don't know at all	I don't care	I usually know	I know
best students	0%	0%	31%	69%
other students	2%	7%	54%	37%
all students	1%	5%	50%	44%

Q13. I'm glad there's a lot of math to learn

not happy at all/not happy/happy/very happy

Responses

	not happy at all	not happy	happy	very happy
best students	6%	25%	59%	10%
other students	24%	54%	16%	6%
all students	20%	47%	26%	7%

Q14. Practical examples help in understanding mathematics

they don't help at all/sometimes they help/they usually help/they always help

Responses

	they don't help at all	sometimes they help	they usually help	they always help
best students	0%	0%	21%	79%
other students	0%	8%	33%	59%
all students	0%	6%	30%	64%

Q15. Learning more math than is really necessary

you have to learn a lot more/you have to learn more/
you have to learn just enough/I would like to learn more

Responses

	you have to learn a lot more	you have to learn more	you have to learn just enough	I would like to learn more
best students	10%	48%	42%	0%
other students	16%	47%	29%	8%
all students	15%	47%	32%	6%

Q16. I will be able to use the mathematics I have learned

I will never use it/I will use it sometimes/I will usually use it/I will always use it

Responses

	I will never use it	I will use it sometimes	I will usually use it	I will always use it
best students	0%	0%	74%	26%
other students	3%	32%	46%	19%
all students	2%	25%	52%	21%

Classify the given things into one of the following categories: concept/mathematical model/physical model/physically existing thing

Q17. triangle
Q18. $\boldsymbol{F = m \cdot a}$
Q19. cat
Q20. amperage
Q21. electric current
Q22. $\boldsymbol{R = \frac{U}{I}}$

Ratio of good responses (questions 17–22)

question	17	18	19	20	21	22
best students	32%	74%	26%	68%	5%	89%
other students	28%	51%	8%	41%	11%	73%
all students	29%	56%	7%	48%	10%	77%

Decide whether the following statements are true or false

Q23. The solution obtained in the mathematical model is always the solution to the technical/practical problem.
Q24. A technical/practical problem can be assigned several different mathematical models.
Q25. Different mathematical models of a technical/practical problem can lead to different solutions.
Q26. To calculate in a mathematical model, you need to understand the technical/practical problem we are modeling.
Q27. All solutions to the technical/practical problem can be obtained from the mathematical model.

Ratio of good responses (questions 23–27)

question	23	24	25	26	27
best students	84%	100%	63%	11%	95%
other students	78%	95%	71%	21%	87%
all students	79%	96%	68%	20%	88%

Appendix E

Table 2 Mathematical knowledge needed to answer the questions and the assessment of the level of knowledge on the basis of the test results.

	substitution into a formula	basic algebra	functions	solving equations	solving systems of equations	calculation with trigonometric functions	vector operations	linear transforms	matrix operations	differentiation	partial differentiation	multivariable extremum problems	integration	extrapolation
ME-1	✓	✓		✓	✓				✓					
ME-2	✓	✓											!	
ME-3	✓	✓		✓		!	!							
ME-4	✓	✓								!				
VM-1		✓	!											!
VM-2	✓	✓				✓								
VM-3	✓	✓											!	
VM-4	✓	✓					!							
L-1	✓	✓		!										
L-2	✓	✓		✓										
L-3	✓	✓		!										
L-4	✓	✓	!	✓	!									
L-5		✓							!	!	o	o		
S-1		✓						!	o					
S-2		✓				✓							✓	
S-3		✓	✓				!			✓				
S-4	✓	✓				✓	!							

In Table 2, S is for Statics. Questions S1-4 are discussed in [10].