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BOOK REVIEWS

Studies in pure mathematics (To the memory of Paul Turán), edited by P. ERDŐS (Editor-in-Chief), L. ALPÁR, G. HALÁSZ and A. SÁRKÖZY, 773 pages, Akadémiai Kiadó, Budapest, and Birkhäuser, Basel, 1983.

From the Editors' preface: "This volume, written by his friends, collaborators and students, is offered to the memory of Paul Turán. Most of the papers they contributed discuss subjects related to his own fields of research. The wide range of topics reflects the versatility of his mathematical activity."

P. ERDŐS (*"Personal reminiscences"*) and G. HALÁSZ (*"Letter to Professor Paul Turán"*) give a characteristic picture of Turán's mathematical style and attitude.

The volume contains 66 scientific papers, written in English (61), French (3) and German (2). In what follows we try to range the papers according to their topics.

Several authors study the *Turán's theory of power sums of complex numbers*, especially the first and second main theorems of Turán. We can read of some generalizations for longer intervals and about the asymptotic behaviour of the factor in the continuous version (G. HALÁSZ), of the second main theorem with the (best possible) constant $4e$ (G. KOLESNIK and E. G. STRAUS), and about pure power sums of complex numbers whose sum is zero (J. SURÁNYI).

Analytic number theory is represented by papers dealing with the zeta function, Dirichlet characters, L-functions and the distribution of primes. We can find improved zero-density estimates (M. JUTILA), approximative Dirichlet-polynomials having zeros with relatively large real part (H. L. MONTGOMERY), essentially improved forms of the oscillatory estimations of the remainder term of the prime number formula (J. PINTZ), inequalities of large sieve type for primes in residue classes (P. D. T. A. ELLIOTT), large sieve extensions of the Brun—Titchmarsh theorem (Y. MOTOHASHI), connections between sieve constants and Siegel's theorem (H. SIEBERT), the average order of Gaussian sums (H. JAGER) and asymptotic formulae for the average of the least positive integer belonging mod p to an exponent divisible by a given prime dividing $p-1$ (K. WIERTELAK).

Algebraic number theory is represented by improved upper bounds for norm form equations and by explicit lower bounds for linear forms with algebraic coefficients (K. GYŐRY and Z. Z. PAPP) and by lower estimates for moduli of polynomials with algebraic integer coefficients at the values of E-functions (A. B. SHIDLOVSKY).

Various problems of *combinatorial number theory* are also discussed. We can refer to distinct sums and products composed from a given sequence (P. ERDŐS and E. SZEMERÉDI), to the behaviour of the product of summands in a representing algorithm (W. NARKIEWICZ), to difference sets, iterations for obtaining an arithmetical progression, density-difference sets (C. L. STEWART and R. TIJDEMAN) and to difference sets with multiple representations (H. L. ABBOT and A. MEIR).

Probabilistic number theory is represented by a sharpened version of the Turán—Kubilius inequality (I. Z. RUZSA), by the joint distribution of the binary digits in multiples of integers (W. M. SCHMIDT) and by the integers representable by subsums of a randomly chosen partition of a positive integer (P. ERDŐS and M. SZALAY).

Separately we mention some topics of the theory of number-theoretical functions. The volume contains asymptotic formulae on generalized divisor functions where the divisors are restricted to a given sequence (P. ERDŐS and A. SÁRKÖZY), various results on near-by multiplicative functions (E. HEPPNER and W. SCHWARZ) and a characterization of $\log n$ as a completely additive function with a growth condition on certain linear combinations of the values (I. KÁTAI).

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Several problems of uniform distribution are also discussed. One can read about generalized pattern integrals (E. Hlawka), of strong irregularities in the distribution of the sequence $\{n\,\alpha\}$ (V. T. Sós) and about certain sequences $\{a_n\alpha\}$ uniformly distributed mod 1 (Lemma 1 of M. Ajtai, I. Havas and J. Komlós).

In what follows we collect the results which can be included in several limit areae of *algebra*. As to the statistical theory of semigroups and groups, the reader can obtain informations about the asymptotic distribution of the order of elements in alternating semigroups and in partial transformation semigroups (B. Harris) and about the order of a randomly chosen element of a Sylow p-subgroup of the symmetric group on p^n letters (P. P. Pálfy and M. Szalay). We can read of infinite Abelian groups with bad topologies (M. Ajtai, I. Havas and J. Komlós), of a sharp quantitative form of the difference between discrete (Fuchsian) and non-discrete groups (Ch. Pommerenke and N. Purzitsky), about the field of definition of the Neron—Severi group (H. P. F. Swinnerton-Dyer) and about bounds on the number of states of a one symbol deterministic automaton needed for the automaton to be equivalent to an arbitrary nondeterministic automaton on n states (J. Dénes, K. H. Kim and F. W. Roush).

Polynomials are discussed in different respects. We can find a lower bound for the product of the values max $(1,|\alpha|)$ over the zeros α of a monic polynomial with integral coefficients — in terms of the number of non-zero coefficients (E. Dobrowolski, W. Lawton and A. Schinzel), extremal polynomials (Á. Elbert), polynomials with curved majorants (R. Pierre and Q. I. Rahman), generalized ultraspherical polynomials (R. Askey and M. E.-H. Ismail), Laguerre entire functions and Turán inequalities (L. Iliev). As to the theory of *interpolation* and *approximation*, we can refer to orthogonal polynomials and rational approximation of holomorphic functions (T. Ganelius), to the close relation between the theorem of Budan—Fourier and lacunary interpolation (G. G. Lorentz) and to the connections between the convergence behaviours of the Lagrange and Hermite—Fejér interpolations (P. Vértesi).

Many papers investigate various topics of *analysis*. One can read about the spherical derivative of meromorphic functions with relatively few poles (J. M. Anderson and J. Clunie), about the growth of meromorphic functions on rays (W. H. J. Fuchs), of the behaviour of a power series on the periphery of its convergence circle (L. Alpár; K.-H. Indlekofer), of gap power series (S. M. Shah; P. Szüsz), about Tauberian theorems (R. C. Vaughan), of entire functions and their derivative on an asymptotic arc (D. Gaier and B. Kjellberg), about Ehrling's problem concerning the coefficients of rational functions (G. Somorjai), about the average behaviour of certain partial sums of the Fourier series of bounded functions (J.-P. Kahane and Y. Katznelson; L. Carleson), of a monotonicity property of some Hausdorff transforms of certain Fourier series (L. Lorch and D. J. Newman), of a Blaschke product with a level-set of infinite length (C. Belna and G. Piranian), of an estimation of type "Schwarz's lemma" for certain functions of several complex variables (M. Waldschmidt), about the minimum of a subharmonic function on a connected set (W. K. Hayman and B. Kjellberg) and of a quasi-Monte Carlo method for approximating the extreme values of a function (H. Niederreiter).

Topology is represented by zero-set spaces as special syntopogenous spaces (Á. Császár).

Geometry is represented by a simple proof of a theorem of Poncelet (I. J. Schoenberg).

Several papers study various problems of the *theory of graphs*. We can refer to a generalization of Turán's graph theorem by stars of vertices (P. Erdős and V. T. Sós), to extremal 3-graphs (W. G. Brown), to k-graphs not having $k+1$ independent vertices (K. H. Kim and F. W. Roush), to extremal graph problems and graph products (M. Simonovits), to the number of complete subgraphs of a graph (L. Lovász and M. Simonovits), to complete bipartite subgraphs contained in spanning tree complements (B. Bollobás, F. R. K. Chung and R. L. Graham), to the decomposition of graphs into complete bipartite subgraphs (F. R. K. Chung, P. Erdős and J. Spencer), to the size Ramsey number of a small graph (F. Harary and Z. Miller) and to a sharp upper bound for the number of simple eigenvalues of the adjacency matrix of a graph having transitive automorphism group (H. Sachs and M. Stiebitz).

M. Szalay (Budapest)