Periodica Mathematica Hungarica Vol. 20 (1), (1989), pp. 93—96

BOOK REVIEWS

A. T. Fomenko, D. B. Fuchs and **V. L. Gutenmacher, Homotopic topology,** 310 pages, Akadémiai Kiadó, Budapest, 1986.

The original version of the book was written on the basis of lectures given at the Moscow State University during the mid-sixties. It begins with the basic notions of homotopic topology.

In *Chapter I* the reader can find the fundamental definitions of homotopy theory. For *CW* complexes the cellular approximation theorem is proved with the help of simplicial complexes. The fundamental group is considered separately, also with some concrete computations. After introducing homotopy groups and covering spaces, the so-called locally-trivial fibration is studied. Any locally trivial fibration has the covering homotopy property (except the uniqueness), which gives rise to the Serre fibration. The relative homotopy groups and the exact sequence of a fibration are introduced next. Beside the Freudenthal theorem on the suspension homomorphism of the sphere, the Whitehead theorem on *CW* complexes, and the Eilenberg—MacLane complexes are also introduced.

Chapter II deals with homology and cohomology theory. The connection between homology and homotopy is studied and the Hurewicz and Whitehead theorems are proved. The universal coefficient formula and the Künneth formula are stated without proof. With the help of the ring structure on the cohomology space, the Hopf invariant is defined. At the end of this chapter obstruction theory is developed.

In *Chapter III* spectral sequences are introduced with many examples. The spectral sequence of a Serre fibration is studied in detail. As an application, the homology groups of $SU(n)$ are computed. A detailed study of the Leray spectral sequence follows, with additions to Leray's theorem. The most interesting addendum includes the definition of transgression, a notion playing an important role in the theory of fibred spaces and in the cohomology theory of compact Lie groups. The obstruction to the cross-section of a fibration is interpreted by the differentials of the spectral sequence. The multiplicative structure of the spectral sequence is illustrated by the complex and real projective spaces. The method of killing spaces is demonstrated on the computation of $\pi_4(S^3)$. In order to compute homotopy groups, one must know the cohomology groups of Eilenberg—MacLane spaces. These groups are first computed over the rational numbers, and corollaries like the Cartan—Serre theorem are derived. The rings $H^*(K(\pi, n); \mathbf{Z}_p)$ are calculated (p is prime) and the results are applied to the computation of the homotopy groups of spheres.

Chapter IV deals with cohomology operations. After the general definitions and examples, the Steenrod algebra is defined. Then Steenrod squares are constructed; their existence is proved by the spectral methods. Adem relations are studied with the help of thė Borel theorem. The cohomology groups of the Eilenberg—MacLane complexes are investigated mod 2; the results are also formulated for mod p. As an application, th e stable homotopy groups of spheres are computed.

Chapter V studies the deepest notion of homotopy theory, the Adams spectral sequence. In the introduction the motivations for the construction are given. After some auxiliary material from homological algebra, the detailed construction of the spectral sequence follows. The Adams spectral sequence has no multiplicative structure of any use, but under certain assumptions it is possible to construct some analogue that turns the Adams spectral sequence into a sequence of rings when X is a sphere. This is also

Akadémiai Kiadó, Budapest
Kluwer Academic Publishers, Dordrecht

discussed in Chapter V, and the rings $\pi_{n+k}(S^n)$ mod 2 are computed up to $k = 7$. The notion of partial operation is the fundamental underlying to the method of Adams. Connection between secondary operations and the second differential in the Adams spectral sequence are discussed, and the natural Postnikov systems are indicated as well. This last chapter ends with a survey of Adams' results on J-homomorphism.

The book is an excellent text on algebraic topology. The presentation is very clear, it contains numerous problems and examples, which help to understand the big material being covered. In addition to explaining difficult concepts, the authors also provide motivations for it. The book is fully illustrated by A. Fomenko's pictures, which give an artistic expression of the material, and make the book pleasant to read.

A. Fialowski (Budapest)

Melwyn W. Jeter, Mathematical programming — An introduction to optimization (Monographs and Textbooks in Pure and Applied Mathematics, 102), ix+342 pages, M. Dekker, New York, 1986.

This book is an elementary introduction into some fundamental topics of mathematical programming. Some basic problems and methods of linear programming, network programming and nonlinear programming are contained in the ten chapters of Jeter's book.

The first, introductory chapter, like the most of the books in this area, gives a few example for mathematical programming problems. The significance of parametrization and post-optimal analysis are shown as well. Historical comments close the introduction.

The second part gives a brief summary of elementary linear algebra. All the results and theorems of linear algebra are presented here which are necessary to develop and understand the theory and algorithms of linear programming.

The primal simplex method and the two phase procedure are the subject of the third chapter. The simplex method is presented in tableau form but geometrical interpretation is discussed as well.

Duality theory of linear programming and some further technics are in the fourth chapter. It starts with duality theory, post-optimal analysis; then the dual simplex method and the linear complementarity problem follow. Lemke's complementary pivoting algorithm closes this section.

Numerical problems and technics for implementing the simplex method are considered in the fifth chapter. The revised simplex method and two methods (product form and elimination form) are presented for calculating the inverse base. The primal-dual algorithm, parametric linear programming and degeneracy (cycling) are considered as well. A few comments on the possibility of decomposition close the linear programming part of the book.

The sixth chapter is a very quick insight into the theory of network programming. Three basic technics, algorithms (labelling technique, simplex specialization, primal-dual approach) are shown on some basic problems of network programming (maximal flow, transshipment problem).

The last four chapters are devoted to nonlinear programming.

A short survey of the theory of convex analysis is given in the seventh chapter. Basic properties of convex functions, convex sets, differential calculus and related topics are considered here in order to give a foundation for further studies.

Perhaps the most important results, the optimality conditions of nonlinear programming are presented in the eighth chapter. The unconstrained case, the constrained case with equality and inequality conditions and nonnegative variables are considered as well. Finally the Kuhn—Tucker theorem is proved.

Unconstrained optimization methods are developed in the ninth chapter. The most important search techniques (Fibonacci, golden section, curve fitting) for one-dimensional optimization are presented. Finally some methods for multidimensional optimization are shown.

The last chapter is devoted to the penalty function methods of constrained optimization. Barrier functions and quadratic penalty function methods are presented.

The author claims that his goal was to write a book for students with relatively small mathematical background (one semester linear algebra and a few analysis). He keeps this goal and these constraints before his eyes all the time. The presentation of the book is correct. The hard and long proofs are left out quite often, only theorems and algorithms