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OSCILLATIONS OF CERTAIN SECOND ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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Abstract

A second order nonlinear differential equation

(*) $$(py')' + q(t)f(z(y), py') = 0$$

is considered. By introducing a Riccati inequality we develop some oscillation criteria for the equation (*).

1. Introduction

I. Bihari ([1], [2]) and Á. Elbert ([3], [4]) considered a kind of second order differential equation called half linear of the form

(1) $$(py')' + q(t)f(y, py') = 0$$

If $f(y, z) \equiv y$, then (1) reduces to

(2) $$(py')' + q(t)y = 0.$$

There are numerous oscillation criteria for Eq.(2). The well known Riccati technique plays an important role in the investigation of oscillation for (2). Elbert has shown that Riccati technique is also useful for such an investigation for Eq.(1). Our purpose in this paper is to use Elbert's approach to study a more general nonlinear equation of the form

(3) $$(py')' + q(t)f(z(y), py') = 0$$

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In case $f(x,y) = x$, then (3) takes form

$$(py')' + q(t)z(y) = 0.$$

In what follows we assume that:
(A_1) f is defined and continuous on $\Omega = R \times R_0$, $R_0 = R - \{0\}$;
(A_2) $xf(x,y) > 0$, for $xy \neq 0$ and $f(0,y) = 0$;
(A_3) $f(\lambda x, \lambda y) = \lambda f(x,y)$ for $\lambda \in (0,\infty), (x,y) \in \Omega$;
(A_4) f is sufficiently smooth to ensure the uniqueness and continuous dependence of solutions on initial conditions;
(A_5) for $F_+(r) = rf(r,1)$ and $F_-(r) = -rf(-r,-1)$,

$$\int_{-\infty}^{\infty} \frac{dv}{1+F_\sigma(r)} < \infty, \lim_{|u|\to\infty} F_\sigma(u) = +\infty \text{ for } \sigma \in \{+,-\}$$

(A_6) p, q are continuous, and $p(t) > 0$;
(A_7) z is continuously differentiable, $z(-u) = z(u), z(u) > 0$ for $u > 0$ and $z'(u) \geq \epsilon > 0$ for some ϵ.

We note that for $z(y) \equiv y$ Eq.(1) is a special case of Eq.(3).

2. Main results

First of all we establish a generalized Riccati inequality for Eq.(3).

LEMMA 2.1. *Assume that conditions (A_1)–(A_7) hold, and $y(t)$ is an eventually positive solution of Eq.(3). Let*

$$u(t) = \frac{p(t)y'(t)}{z(y(t))}. \tag{4}$$

Then

$$S'(t) + \frac{H(S)}{p(t)} + q(t) \leq 0, \tag{5}$$

where

$$g(u) = \begin{cases} \int_{\frac{1}{u}}^{\infty} \frac{dv}{F_+(v)}, & \text{if } u > 0, \\ -\int_{-\infty}^{\frac{1}{u}} \frac{dv}{F_-(v)}, & \text{if } u < 0, \\ g(0) = 0, \end{cases} \tag{6}$$

$$S = S(t) = g(u(t)), \tag{7}$$

and