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BE n FIXED, OR GO TO INFINITY: DIFFICULTIES ARE WELCOME, AND ALMOST SURELY OVERCOME

A TRIBUTE TO ENDRE CSÁKI

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Preamble

The immediate aim of these introductory lines is to provide some initial global guidelines for gaining a quick view of the complex milieu of Endre Csáki's work in probability and mathematical statistics. More will be said on some of the items mentioned here, in the main body of this exposition.

First we mention the topics that illustrate the main themes of his manifold research in these two areas of stochastics, and note also that he has achieved fundamental new results in each of the following areas: classical random walks, empirical distributions and processes, Brownian motion (Wiener process) and related Gaussian processes, local times, additive functionals, iterated processes, almost sure central limit theorems, Hilbert transform and Cauchy principal values of stochastic processes.

When talking about contributions to the topic of classical random walks, W. Feller's classical first volume can serve in lieu of a definition and, in this regard, we have the impressive set of papers 2., 3., 5., 6., 7., 24., 43., 44., 54., 67., 77., 78., 81. and 97. of Endre Csáki's list of publications in mind.

Concerning empirical distributions and processes, we first call attention to the expositions 14., 15. and 17., which deserve special mention, and can be viewed as continuation of Alfréd Rényi's pioneering work in this regard. In addition to these works being first time ground breaking contributions to studying the almost sure asymptotic behaviour of weighted empirical processes, due to their precision and depth, they have also become well known as classical gems in this area of limit theorems. These results of Endre Csáki, and his subsequent ones along these lines, like for example the expositions 27., 28., 57., 68. and 87. of his list of publications, have

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had a strong influence on the international literature and have played pivotal roles when dealing with this important topic of interest (cf., e.g., the books M. Csörgő–P. Révész: *Strong Approximations in Probability and Statistics* (Academic Press, 1981), J. H. J. Einmahl: *Multivariate Empirical Processes* (Amsterdam, 1986), G. Shorack–J. Wellner: *Empirical Processes with Applications to Statistics* (Wiley, 1986), M. Csörgő–L. Horváth: *Weighted Approximations in Probability and Statistics* (Wiley, 1993)).

Brownian motion (Wiener process), its local time and its other additive functionals play a central role in the theory and applications of stochastic processes. The area of stochastic analysis has been greatly enriched in the last few years by studying Hilbert transform and Cauchy principal values of stochastic processes. The study of the fine analytic properties of the trajectories of these processes that was initiated by Paul Lévy in the thirties constitutes one of the most important topics of stochastic analysis and, on account of its richness in interesting problems, it continues to be a very popular one in these days as well. One of the most seminal contributions to this area of stochastics is the celebrated Erdős–Rényi law of large numbers of 1971. This result and the investigation and utilization of its relation to Paul Lévy's earlier work has led to many of the significant flourishes of the Hungarian probability school. The papers 19., 20., 22., 25., 26., 29., 30., 31., 33., 34., 35., 36., 37., 38., 39., 41., 42., 45., 47., 49., 50., 51., 52., 53., 55., 63., 65., 71., 74., 86., 88., 90., 92., 93., 94., 95., 96., 98., 99., 101., 102., 104. and 105. of Endre Csáki's list of publications belong to the naturally interacting categories that are mentioned in this paragraph. They clearly underline also his international reputation as one of the leading researchers in these areas of eminence of the Hungarian school. The mentioned articles contain a large number of new and path-breaking results, and many of them also play fundamental roles in the book, Pál Révész: *Random Walk in Random and Non-Random Environments* (World Scientific, 1990). Outstanding even of these are: 25., which creates a surprising connection between Strassen's and Chung's law of the iterated logarithm which, in turn has led to further investigations of this phenomenon by de Acosta, Grill, Kuelbs, Wenbo Li, Talagrand; 29. with Pál Révész, in which they prove a significantly new invariance principle for approximating the local time of a rather general random walk by that of a Wiener process; 41. with Antónia Földes, in which they establish an integral criteria for the lower classes of the maximal local time of a Wiener process that, in particular, also results in identifying an determining a constant that first appeared in an earlier work of Kesten in this regard.

Paper 52., in which the authors study the difference of Brownian local time from its zero place via a Gaussian random field, has in a natural way led to investigating iterated processes as a new field on its own, an area that is now investigated by many in the literature. Endre Csáki's fundamental results in this new area are contained in the papers 72., 75., 79, 85. and 100. of his list.

One of the most important diffusion processes is the stationary Gauss–Markov process, the so-called Ornstein–Uhlenbeck process, whose infinite dimensional version has been studied by many in the last 10 or so years on account of its importance.