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RANDOM WALK AND BROWNIAN LOCAL TIMES IN WIENER SHEETS: A TRIBUTE TO MY ALMOST SURELY MOST VISITED 75 YEARS YOUNG BEST FRIENDS, ENDRE CSÁKI AND PÁL RÉVÉSZ

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Dedicated to Endre Csáki and Pál Révész on the occasion of their 75th birthdays

Preamble

Many of us from all over the world and Hungary were happily gathered in Budapest on November 5–7, 2009 to celebrate the 75^{th} birthdays of Endre Csáki and Pál Révész, attending an International Conference in their honour in the Alfréd Rényi Institute of Mathematics of the Hungarian Academy of Sciences. It pleases me very much to also have the opportunity to contribute to this Festschrift Volume of Periodica Mathematica Hungarica that is published in honour of these truly eminent scholars, who also happen to have been two of my most favourite cherished friends for a long time now. I am also glad to say that I already had the privilege to pay tribute to their achievements separately ([9], [10]), when they turned 65, and also on the occasion of celebrating them together on their 70^{th} birthday ([11]).

I note in passing that reference numbers preceded by Cs in [ ] will refer to Endre Csáki's List of Publications, while those preceded by R in [ ] to Pál Révész's List of Publications. References in [ ] refer to those at the end of this exposition.

Back to 2004/2005, in [11] I concentrated mainly on highlighting some of the peaks of their formidable collaboration since 1979, when they published the first one

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of the then twenty-four joint papers together and, frequently, with others as well. Since that time the latter number has already grown one and a half times to thirty-six, and there are several more in progress. The larger milieu of this collaboration is nicely described by Antónia Földes in the Introduction of her 2005 tribute [16] to Endre Csáki and Pál Révész. In it, she provides a personal overview of those papers between 1987 and 2004 which were solely written by the three of them, while in her present contribution to this volume [17], she gives an insightful account of the most recent collaborative work of the four of us, Csáki et al., [Cs132], [Cs136], [R185], [R188], [8], concerning random walking on a 2-dimensional comb lattice that is obtained from $\mathbb{Z}^2$ by removing all horizontal edges off the x-axis. Backtracking a bit again, I would also like to mention the paper [Cs118] by Endre Csáki, Antónia Földes and Zhan Shi, which they wrote in honour of my 70^{th} birthday, reviewing our joint work, and thus, inevitably, some of that with Pál Révész as well. Yet another informative exposition in this regard is by Lajos Horváth and Barbara Szyszkowicz, eds. [19], who review path properties of my by then forty or so years of research in probability and statistics that I have been lucky enough to be able to continue doing ever since.

In Section 1 of this exposition I concentrate on presenting asymptotic properties of random walk local times that can be studied via invariance principles in terms of Brownian local times, while in Section 2 the emphasis is on the parallel dichotomous asymptotic behaviour of these local times in terms of their additive functionals. There is an inevitable interplay of the notions of invariance and non-invariance that helps to explain both in a historical context that I adhere to. These goals are frequently achieved via studying the asymptotic properties of appropriate iterated processes whose local and occupation times are the subject of Section 3, and that too rendered in a historical context. To illustrate the intrinsic nature of our studies in this area of stochastics, I quote here the Abstract of the latest one of them, Csáki et al. [Cs136], [R188] of 2010, that reads as follows: "We obtain upper and lower class integral tests for the space-wise supremum of the iterated local time of two independent Wiener processes. We then establish a strong invariance principle between this iterated local time and the local time process of the simple symmetric random walk on the two-dimensional comb lattice. The latter, in turn, enables us to conclude upper and lower class tests for the local time of simple symmetric random walk on the two-dimensional comb lattice as well."

A glimpse of some of these results can be seen in Section 4 of Antónia Földes [17] in this volume, while Section 3 of my tribute can be viewed as some of the preliminary excursions along these lines.

Though the Csáki et al. papers [Cs136], [R188] bring us up to publications already in 2010, I wish to note that in this exposition I have ended up not even trying to fully explore, and give adequate credit to, the new dimensions of the rich stochastic world of Endre Csáki and Pál Révész that has been created since my